Module-5
Limit State Design of Columns and Footings

Analysis and design of short axially loaded RC column. Design of columns with uniaxial and biaxial moments, Design concepts of the footings. Design of Rectangular and square column footings with axial load and also for axial load & moment.

---10 hrs

UNIT:1-Limit State Design of Columns

INTRODUCTION

A compression member is a structural element which is subjected to axial compressive forces. Compression members are most commonly encountered in Reinforced concrete buildings as columns (and sometimes as Reinforced concrete walls).

A column supports the loads of super-structure and transfers them to or to the foundation below it. It is basically a compression member since loads are acting along its longitudinal axis. There may be bending moment due to wind, earthquake or accidental loads. Bending of a column is also due to monolithic construction and eccentricity of loads from superstructure. Being a compression member, its analysis and design becomes primary a function of length to lateral dimension ratio (slenderness ratio). When length of a compression member is too short (a pedestal), its design and detailing differ from that of a column.

- **Column:** A column is defined as a vertical compression member which is mainly subjected to axial loads and the effective length of which exceeds three times its least lateral dimension.
- **Pedestal:** The compression member whose effective length is less than three times its least lateral dimension is called as Pedestal.
- **Strut:** The compression member which is inclined or horizontal and is subjected to axial load is called as Strut. Struts are used in trusses.

. CLASSIFICATION OF COLUMNS

A column may be classified based on different criteria such as:

1. Classification of columns based on type of loading
2. Classification of columns based on the slenderness ratio
3. Classification of columns based on shape
4. Classification of columns based on materials

CLASSIFICATION OF COLUMNS BASED ON TYPE OF LOADING

A column may be classified as follows, based on type of loading:

a) Axially loaded columns (Concentric loading)

b) Eccentrically loaded columns(Uniaxial bending or Biaxial bending)

a) **Axially loaded Columns :** (Concentric loading) The columns which are subjected to loads acting along the longitudinal axis or centroid of the column section are called as axially loaded columns as shown in fig. 5.1(a). Axially loaded column is subjected to direct compressive stress only and no bending stress develops anywhere in the column section.

b) **Eccentrically loaded Columns:** Eccentrically loaded columns are those columns in which the loads do not act on the longitudinal axis of the column. They are subjected to direct compressive stress and bending stress both. Eccentrically loaded columns may be subjected to uniaxial bending as shown in fig. 5.1(b) or biaxial bending as shown in fig. 5.1(c) depending upon the line of action of load with respect to the two axis of the column.

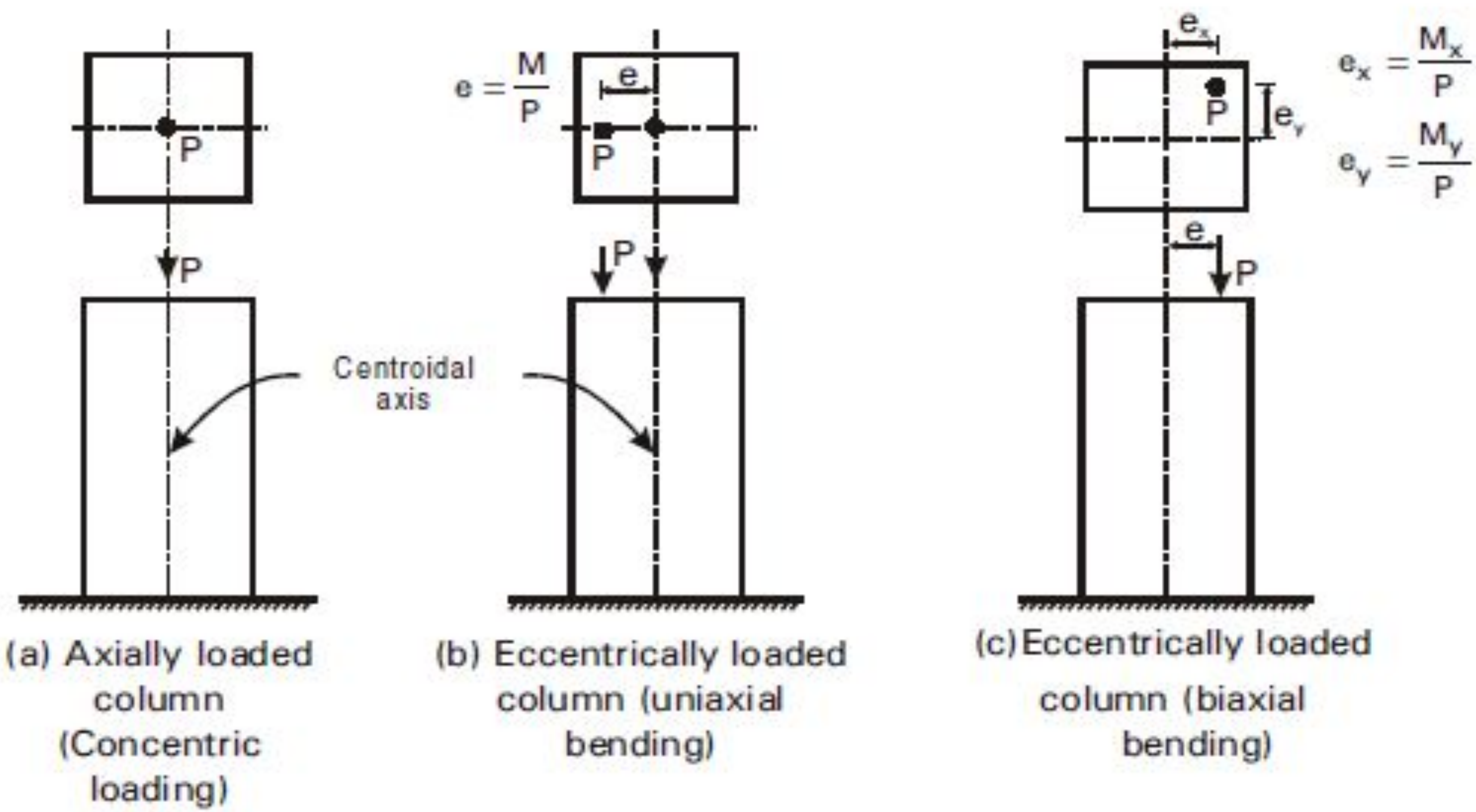


Fig. 5.1 : Different loading situations in columns

. CLASSIFICATION OF COLUMNS BASED ON THE SLENDERNESS RATIO

A column may be classified as follows, based on value of slenderness ratio (As per IS 456:2000 clause 25.1.2):

- Short Columns
- Long Columns

Short Columns: The column is considered as short when the slenderness ratio of column i.e., ratio of effective length to its least lateral dimension is less than or equal to 12.

$$\text{If } \frac{l_{ex}}{D} \quad \text{or} \quad \frac{l_{ey}}{b} < 12 \rightarrow \text{short coulmn}$$

Long Columns: The column is considered as long or slender column when the slenderness ratio of column i.e., ratio of effective length to its least lateral dimension is greater than 12.

$$\text{If } \frac{l_{ex}}{D} \quad \text{or} \quad \frac{l_{ey}}{b} > 12 \rightarrow \text{short coulmn}$$

Where,

l_{ex} = Effective length in respect of the major axis

D = Depth in respect of the major axis

l_{ey} = Effective length in respect to minor axis

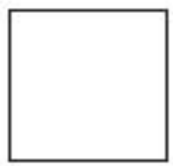
b = Width of the member

, Classification of Columns based on shape

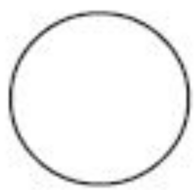
A column may be classified as follows, based on shape

1. Rectangular

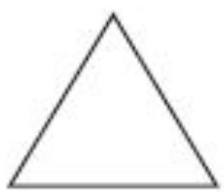
2. Square



3. Circular

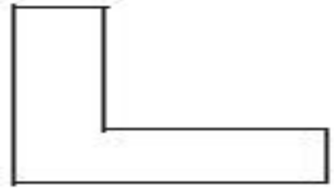


4. Polygon

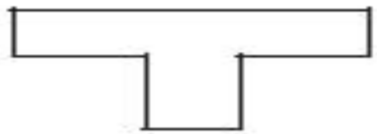


or

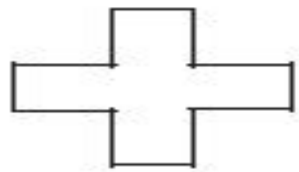
5. L - typed



6. T - type



7. Plus - type



Classification of Columns based on materials

A column may be classified as follows, based on materials

1. RCC Columns

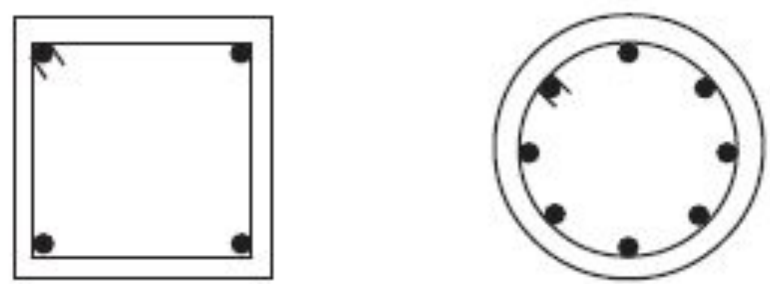


Fig. 5.3 : RCC square and circular columns with steel reinforcements

2. Steel Columns

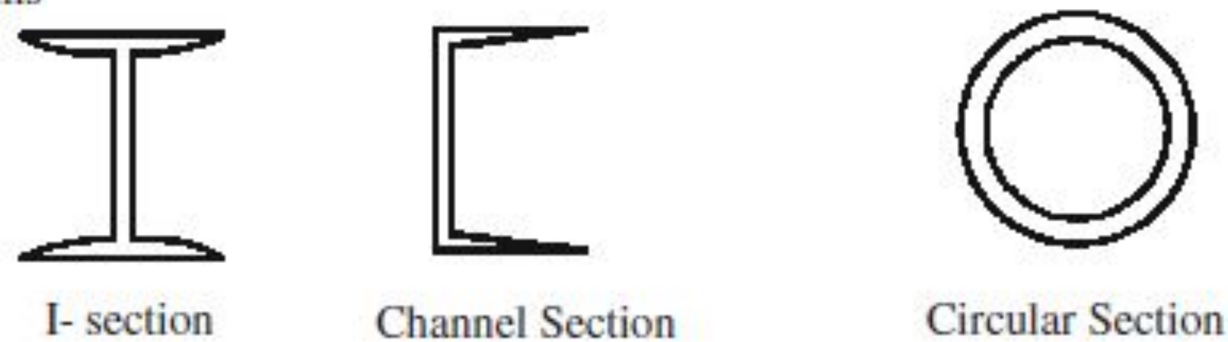


Fig. 5.4 Steel columns

3. Composite Columns

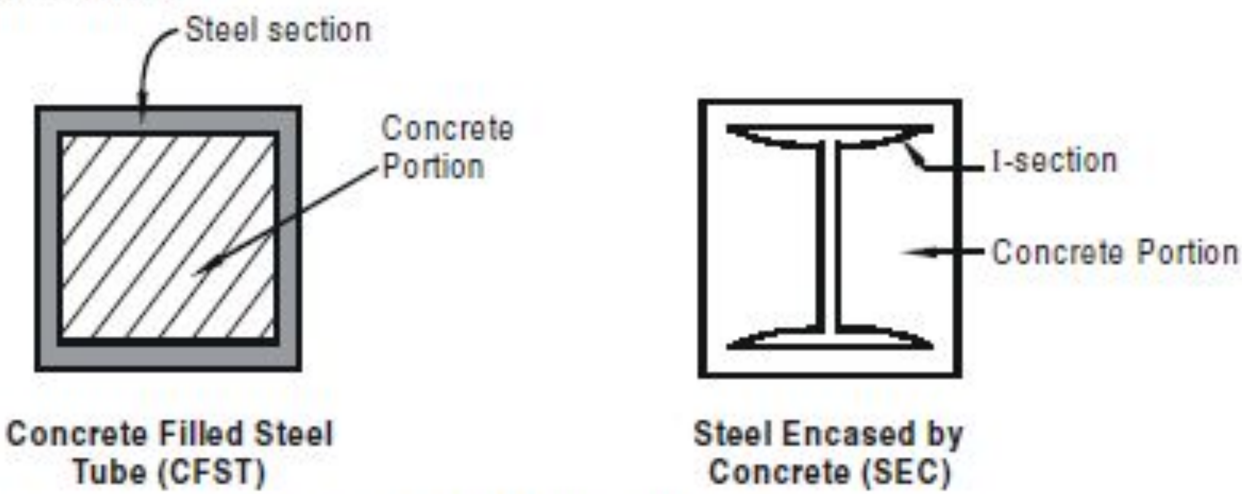


Fig. 5.5 Composite columns

COMPARISON BETWEEN SHORT COLUMN AND LONG COLUMN

Sl. No.	Short column	Long column
1	The column is considered as short when ratio of effective length to its least lateral dimension is less than or equal to 12	The column is considered as long or slender column when the ratio of effective length to its least lateral dimension is greater than 12.
2	The ratio of effective length of a column to its least radius of gyration is less than or equal to 40.	The ratio of effective length of a column to its least radius of gyration is greater than 40.
3	Buckling tendency is very low.	Long columns buckle easily.
4	The load carrying capacity is high as compared to long column of the same cross-sectional area.	The load carrying capacity of a long column is less as compared to short column of same cross-sectional area.
5	The failure of the short column is by crushing.	The column generally fails in buckling.

ASSUMPTIONS MADE FOR THE LIMIT STATE OF COLLAPSE IN COMPRESSION

The following assumptions are made for the limit state of collapse in compression

i) Plane sections normal to the axis remain plane after bending.

ii) The relationship between stress-strain distribution in concrete is assumed to be parabolic. The maximum compressive stress is equal to $0.67 f_{ck}/1.5$ or $0.446 f_{ck}$.

iii) The tensile strength of concrete is ignored.

iv) The stresses in reinforcement are derived from the representative stress-strain curve for the type of steel used.

v) The maximum compressive strain in concrete in axial compression is taken as 0.002.

vi) The maximum compression strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, but when there is no tension on the section, is taken as 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

vii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, when part of the section is in tension, is taken as 0.0035.

SHORT AXIALLY LOADED RC COLUMN

As per IS 456:2000, clause 39.3

The members may be designed by the following equation.

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Where, P_u = Factored Axial on the member

f_{ck} = Characteristic compressive strength of the concrete

f_y = Characteristic strength of the compression reinforcement

A_g = Gross cross-sectional area (Total area)

i.e., $A_g = A_c + A_{sc}$

A_c = Area of concrete

A_{sc} = Area of longitudinal reinforcement for columns

Note: The column members are to be designed for a minimum eccentricity of the load in two principle directions.

STEPS FOR SOLVING PROBLEMS ON SHORT AXIALLY LOADED RC COLUMNS

Type I Problems

Given: Size of column($b \times D$), Area of longitudinal reinforcement(A_{sc}) or No.of bars with diameter of bars, Grade of concrete and steel.

Required: Axial load (P) or ultimate load (P_u).

Steps:

1. Calculate area of column, $A_g = b \times D$

2. Calculate area of longitudinal reinforcement, $A_{sc} = \text{No. of bars} \times \frac{\pi \times \phi_L^2}{4}$

   [ϕ_L = diameter of longitudinal bars]

3. Area of concrete, $A_c = A_g - A_{sc}$ [$\because A_g = A_c + A_{sc}$]

4. Factored Axial load (Refer IS: 456-2000, clause 39.3)

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Service or Working axial load, $P = \frac{P_u}{1.5}$

Type II Problems

Given: Size of column($b \times D$), Axial load (P), Grade of concrete and steel.

Required: Area of longitudinal reinforcement (A_{sc}) and design of lateral ties.

Steps:

1. Calculate factored axial load $P_u = 1.5 \times P$
2. Area of column, $A_g = b \times D$
3. Area of concrete, $A_c = A_g - A_{sc}$
4. Design of Longitudinal Reinforcement

   Factored Axial load,

   $$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

   Substitute value of A_c as $(A_g - A_{sc})$

   $$P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc}$$

   Determine A_{sc} as all other terms are known.

   Assume diameter of bars (ϕ_L) (e.g: ϕ_L = 12mm, 16mm, 20mm, 25mm, 28mm, 32mm,….)

   Area of one bar, $a_{sc} = \dfrac{\pi \times \phi_L^2}{4}$

   No. of bars $= \dfrac{A_{sc}}{a_{sc}}$ (Round off to even number, minimum 4 bars for square or rectangular and 6 bars for circular column)

5. Design of Lateral ties

➢ Diameter : Greater of the following

i) $\frac{\phi_L}{4}$ ii) 6mm

➢ Pitch (Refer IS: 456-2000, Clause 26.5.2-c):

Spacing is least of the following

i) Least Lateral dimension : b or D (least size of column)

ii) 16 times the smallest diameter of longitudinal bar (ϕ_L)

iii) 300mm

6. Column reinforcement details (Longitudinal section and cross section)

Type III Problem (Design of short axially loaded RC column)

Given: Axial load, Grade of concrete and steel

Required: Size of column, Design of longitudinal and lateral reinforcement (transverse reinforcement)

Steps

1. Calculate, $P_u = 1.5 \times P$
2. Area of Steel,

Assume % of steel as $0.8\,\% \times A_g$ (0.8% to 4%) or some times it may be given in problem.

$$\therefore A_{sc} = \frac{0.8}{100} \times A_g = 0.008\, A_g$$

3. Area of concrete, $A_c = A_g - A_{sc}$
4. Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Substitute the value of 'A_c' and A_{sc} in above expression, calculate 'A_g'.

(i) If square column to be designed ($A_g = b \times D$), then size of column, $b = D = \sqrt{A_g}$

(ii) If rectangular column is to be designed ($A_g = b \times D$), Assume b or D and then calculate the size of column i.e., $b = \dfrac{A_g}{D}$ or $D = \dfrac{A_g}{b}$

(iii) If circular column is to be designed ($A_g = \frac{\pi D^2}{4}$), $\therefore$ Diameter, $D = \sqrt{\frac{4A_g}{\pi}}$

5. Design of Longitudinal reinforcement

   W.K.T, $A_{sc} = 0.008\ A_g$

   Assume suitable diameter of longitudinal bars (ϕ_L)

   $\therefore$ Area of one bar, $a_{sc} = \frac{\pi \times \phi_L^2}{4}$

   $\Rightarrow$ No. of bars $= \frac{A_{sc}}{a_{sc}}$

6. Design of Lateral ties

   Same as in the type II problem

7. Column reinforcement details (L/S and C/S)

Type IV Problem (Design of short axially loaded RC column)

Given: Axial load, Grade of concrete and steel, unsupported length (l) with end conditions.

Required: Size of column, Effective length (l_{eff}), check for slenderness ratio and minimum eccentricity, Design of longitudinal and lateral reinforcement.

Steps

1. Calculate, $P_u = 1.5 \times P$
2. Area of Steel (Same as step 2 - Type III)
3. Area of concrete (Same as step 3 - Type III)
4. Determine the size of column (Same as Step 4 - Type III)
5. Check for slenderness ratio
   - Calculate Effective length (l_{eff}) based on end conditions.
   - Check, $\frac{l_{eff}}{b} < 12$ or $\frac{l_{eff}}{D} < 12 \rightarrow$ for square column
   - Check, $\frac{l_{eff}}{b} < 12$ and $\frac{l_{eff}}{D} < 12 \rightarrow$ for rectangular column
   - Check, $\frac{l_{eff}}{D} < 12 \rightarrow$ for circular column

6. Check for minimum eccentricity (IS: 456-2000, Clause 25.4)

- Along longer direction,

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20 \text{ mm}$$

$e_{permissible} \leq 0.05\, D$

then, compare e_{min} and $e_{permissible}$

- Along shorter direction,

$$e_{min} = \frac{l}{500} + \frac{b}{30} \geq 20 \text{ mm}$$

$e_{permissible} \leq 0.05\, b$

then, compare e_{min} and $e_{permissible}$

If $e_{min} < e_{permissible}$ → Design is safe (Column can be designed as axially loaded short column)

If $e_{min} > e_{permissible}$ → Design is not safe (Hence increase the size of column)

or

The column is to be designed as uniaxial bending or biaxial bending case

7. Design of longitudinal reinforcement (Same as step 5 - Type III)
8. Design of lateral ties (Same as step 5 - Type III)
9. Reinforcement details (L/S and C/S)

WORKED EXAMPLES ON SHORT AXIALLY LOADED RC COLUMNS

1. **A reinforced concrete short column, 300 mm square is reinforced with 4 bars of 16 mm dia, determine the ultimate load capacity of the column, using M-20 grade concrete and Fe-415 grade steel. What will be the allowable service load?**

Solution:

Given : Size of column = 300 $mm \times 300$ $mm = b \times D$

Longitudinal reinforcement = 4 –16 mm dia bar, $P = ?$

$$M_{20}, \qquad f_{ck} = 20 \text{ N / mm}^2$$

$$\text{Fe}_{415}, \qquad f_y = 415\, N/mm^2 \qquad\qquad \because A_g = b \times D$$

Step:1

Calculate area of column

$$A_g = 300 \times 300 = 90000\text{mm}^2$$

Step:2

Calculate area of longitudinal reinforcement

$$A_{sc} = 4 \times \frac{\pi(16)^2}{4} = 804.2mm^2$$

Step:3

Area of concrete

$$A_c = A_g - A_{sc} = 90000 - 804.2 = 89195.8\text{mm}^2$$

Step:4

Factored axial load

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$P_u = 0.4 \times 20 \times 89195.8 + 0.67 \times 415 \times 804.2$$

$$= 937174.21N = 937.17kN$$

Step:5

Allowable service load $P = \dfrac{P_u}{1.5} = \dfrac{937.17}{1.5} = \mathbf{624.78kN}$

2. A short circular column of diameter 400m is reinforced with 6 numbers of 16 mm dia. Find the axial factored load on the column if M20 grade concrete and Fe415 grade steel is used.

Solution:

Given : $f_{ck} = 20N/mm^2$, $f_y = 415N/mm^2$, $D = 400\text{mm}$, $A_{sc} = 6 - 16\phi$ and $P_u = ?$

Step:1

Area of steel, $A_{sc} = 6 \times \frac{\pi}{4} \times 16^2 = 1206.4mm^2$

Step:2

Area of concrete = $A_c = \text{gross area} - \text{area of steel}$

$$\because A_g = \frac{\pi}{4} D^2$$

$$A_c = A_g - A_{sc}$$

$$= \frac{\pi}{4} \times 400^2 - 1206.4 = 124457.3mm^2$$

Step:3

For axially loaded short columns, factored load is given by

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc}$$

$$= 0.4 \times 20 \times 124457.3 + 0.67 \times 415 \times 1206.4$$

$$P_u = 1331.1 \times 10^3 N = 1331.1\text{kN}$$

3. Design necessary reinforcement for RC short column 400 mm × 600 mm to carry an axial load of 1800 kN. Use M-20 concrete and Fe-415 steel. Sketch the reinforcement details.

Solution:

Given: Axial load, P = 1800 kN, b = 400mm, D = 600mm, f_{ck} = 20N/mm² and $f_y = 415$ N/mm²

Step:1

Factored axial load, $P_u = 1.5 \times 1800 = 2700\ kN$

Step:2

Area of the column, $A_g = 400 \times 600 = 240000 mm^2$

Step:3

Area of concrete $A_c = A_g - A_{sc} = 400 \times 600 - A_{sc} = 34 \times 10^4 - A_{sc}$

Step:4

Design of Longitudinal reinforcement

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2700 \times 10^3 = 0.4 \times 20(24 \times 10^4 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$2700 \times 10^3 = 1.92 \times 10^6 - 8A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = 2888.35 mm^2$$

$$\% \text{ of steel, } p_t = \frac{100 A_{sc}}{bD} = \frac{100 \times 2888.35}{400 \times 600} = 1.2\%$$

Minimum reinforcement = 0.8%

Maximum reinforcement = 4%

$\Rightarrow 0.8 < 1.2\% < 4\% \rightarrow$ It is within limits

Using 20 mm dia bars, No. of bars = $\frac{A_{sc}}{a_{sc}} = \frac{2888.35}{\frac{\pi}{4}(20)^2} = 9.19$ say 10

$\therefore$ Provide 10 bars of 20 mm diameter as longitudinal reinforcement.

Step:5

Design of Lateral ties

- Diameter of Lateral ties: Greater of the following

i) 6mm

ii) $\frac{1}{4} \times 20 = 5$ mm Say 6 mm

➢ Spacing of lateral ties: Least of the following

i) Least lateral dimension, $b = 400$ mm

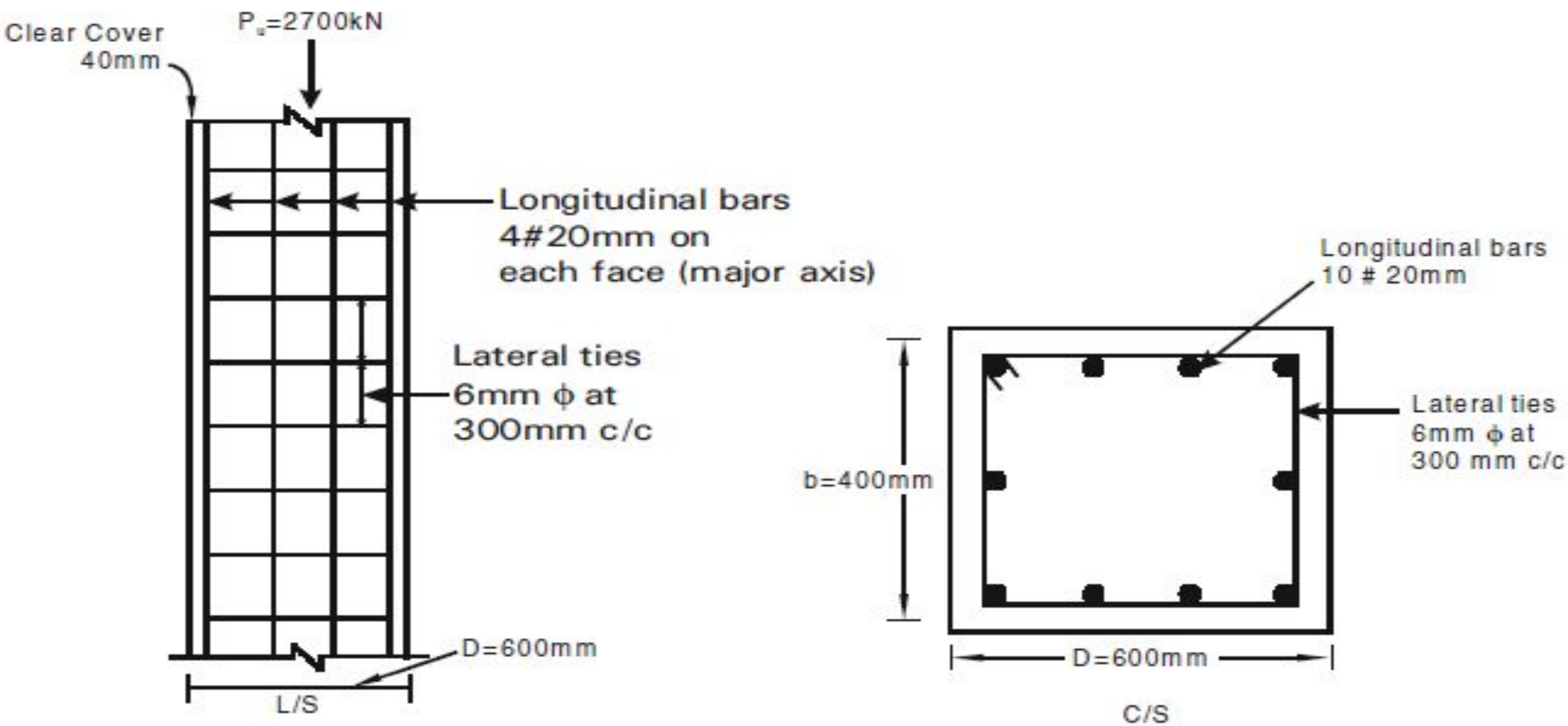
ii) $16 \times$ dia. of main bar $= 16 \times 20 = 320$ mm

iii) 300 mm

$\therefore$ Provide 6 mm dia at 300 mm c/c.

Step:6

Column Reinforcement details (L/S and C/S)



4. Design necessary reinforcement for RC column 400 mm × 600 mm to carry an axial load of 2000 kN. The length of the column is 3 m. use M-20 concrete and Fe-415 steel.

Solution:

Given: Axial load = $P = 2000\ kN$, $f_{ck} = 20\text{N/mm}^2$, $f_y = 415\ \text{N/mm}^2$, $b = 400\text{mm}$, $D = 600\text{mm}$ and $l = 3\text{m}$

Step:1

Factored axial load $P_u = 1.5 \times 2000 = 3000\ kN$

Step:2

Check for slenderness ratio

Assuming both ends hinged, $l = l_{eff} = 3\text{m} = 3000\text{mm}$

$$\frac{l_{eff}}{b} = \frac{3000}{400} = 7.5 < 12$$

$$\frac{l_{eff}}{D} = \frac{3000}{600} = 5 < 12$$

$\therefore$ The column is a short column

Step:3

Check for minimum eccentricity

Note: Minimum eccentricity as per IS code, $e_{min} = 20mm$, clause 25.4

- Along longer direction

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20mm$$

$$= \frac{3000}{500} + \frac{600}{30} = 26mm$$

$e_{permissible} \leq 0.05\ D = 0.05 \times 600 = 30mm,$

$\Rightarrow e_{min} < e_{permissible}$ or $e_{permissible} > e_{min}$

$\therefore$ Design is safe

- Along shorter direction

$$e_{min} = \frac{l}{500} + \frac{b}{30} \geq 20mm$$

$$= \frac{3000}{500} + \frac{400}{30} = 19.33mm$$

$e_{permissible} \leq 0.05b = 0.05 \times 400 = 20mm,$

$\Rightarrow e_{min} < e_{permissible}$, Design is safe

Hence the column can be designed as axially loaded short column

Step:4

Design of Longitudinal reinforcement

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \qquad \text{From IS : 456 - 2000, Clause 39.3}$$

$$A_c = A_g - A_{sc} = 400 \times 600 - A_{sc} = 24 \times 10^4 - A_{sc}$$

$$3000 \times 10^3 = 0.4 \times 20(24 \times 10^4 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$3000 \times 10^3 = 1.92 \times 10^6 - 8A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = 3999.25 mm^2$$

$$\% \text{ of steel, } p_t = \frac{100 A_{sc}}{bD} = \frac{100 \times 3999.25}{400 \times 600} = 1.66\%$$

$$\Rightarrow 0.8\% < 1.66\% < 4\%$$

$\therefore$ It is within limits

Providing 25 mm dia bars

$$\text{No of bars} = \frac{3999.25}{\frac{\pi(25)^2}{4}} = 8.14 \text{ say } 10$$

Provide 10 bars of 25 mm diameter as longitudinal reinforcement.

Step:5

Design of Lateral ties

- Diameter of Lateral ties: Greater of the following

i) 6*mm*

ii) $\frac{1}{4} \times 25 = 6.25$ Say 8 *mm*

- Spacing of lateral ties: Least of the following

i) Least lateral dimension, $b = 400\ mm$

ii) $16 \times$ dia of main bar $= 16 \times 25 = 400\ mm$

iii) 300 *mm*

$\therefore$ Provide 8 mm dia at 300 mm c/c.

5. Design a column of size 450 mm × 600 mm and having 3 m unsupported length. The column is subjected to a load of 2000 kN and is effectively held in position but not restrained against rotation. Use M20 concrete and Fe 415 steel.

Solution:

Given : Axial load = P = 2000 kN, l = 3m, b = 450mm, D = 600mm, $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step:1

Factored axial load, $P_u = 1.5 \times 2000 = 3000$ kN

Step:2

Check for slenderness ratio

For effectively held in position but not restrained against rotation,

Effective length, $l_{eff} = 1.0 \times l$ (As per IS 456:2000, Table 28)

$$\frac{l_{eff}}{b} = \frac{3000}{450} = 6.67 < 12$$

$$\frac{l_{eff}}{D} = \frac{3000}{600} = 5 < 12$$

$\therefore$ The column is a short column.

Step:3

Check for minimum eccentricity

- Along longer direction

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{3000}{500} + \frac{600}{30} = 26mm$$

$$e_{permissible} \leq 0.05\,D = 0.05 \times 600 = 30\text{mm}$$

$$\Rightarrow e_{min} < e_{permissible}, \text{ Design is safe}$$

- Along shorter direction

$$e_{min} = \frac{l}{500} + \frac{b}{30} \geq 20\text{mm}$$

$$= \frac{3000}{500} + \frac{450}{30} = 21mm$$

$$e_{permissible} \leq 0.05\text{b} = 0.05 \times 450 = 22.5\text{mm}$$

$$\Rightarrow e_{min} < e_{permissible}, \text{ Design is safe}$$

Hence the design is done as a short axially loaded column using formula

Step:4

Design of Longitudinal reinforcement

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_c = A_g - A_{sc} = 450 \times 600 - A_{sc} = 27 \times 10^4 - A_{sc}$$

$$3000 \times 10^3 = 0.4 \times 20(27 \times 10^4 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$3000 \times 10^3 = 2.16 \times 10^6 - 8A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = \frac{840000}{270.05} = 3110.53 mm^2$$

$$\% \text{ of steel, } p_t = \frac{100 A_{sc}}{bD} = \frac{100 \times 3110.53}{450 \times 600} = 1.15\%$$

Percentage of steel is more than 0.8% and less than 4%, hence ok.

Providing 25 mm dia bars

$$\text{No. of bars} = \frac{3110.53}{\frac{\pi (25)^2}{4}} = 6.34 \text{ say } 8$$

Provide 8 bars of 25 mm diameter as longitudinal reinforcement.

Step:5

Design of Lateral ties:

- The diameter of lateral ties should be greater of the following:
  a) 6mm
  b) $\frac{1}{4} \times 25 = 6.25$ mm, say 8mm

  Providing 8mm dia lateral ties.
- Spacing of lateral ties should be least of the following
  a) Least lateral dimension $= b = 450$ mm
  b) $16 \times$ dia of main bar $= 16 \times 25 = 400$ mm
  c) 300 mm

  $\therefore$ Provide 8 mm dia at 300 mm c/c

6. Design a circular column of diameter 450 mm subjected to a load of 1200 kN. The column is having lateral ties. The column is 3m long and is effectively held in position at both ends but not restrained against rotation. Use M25 concrete and Fe 415 steel.

Solution:

Given : Axial load = P = 1200 kN, D = 450mm, l = 3m, $f_{ck} = 25\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step:1

Factored axial load, $P_u = 1.5 \times 1200 = 1800$ kN

Step:2

Check for Slenderness ratio

For effectively held in position but not restrained against rotation,

Effective length, $l_{eff} = 1.0l$ (As per IS 456:2000, Table 28)

$$\frac{l_{eff}}{D} = \frac{3000}{450} = 6.67 < 12$$

∴ The column is a short column.

Step:3

Check for minimum eccentricity

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{3000}{500} + \frac{450}{30} = 21\text{mm}$$

$$e_{\text{permissible}} \leq 0.05\,D = 0.05 \times 450 = 22.5\text{mm}$$

$$\Rightarrow e_{\min} < e_{\text{permissible}}, \text{ Design is safe}$$

∴ The design is done as a short axially loaded column using formula

Step:4

Design of Longitudinal reinforcement

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Area of column, $A_g = \dfrac{\pi D^2}{4} = \dfrac{\pi \times 450^2}{4} = 159043\text{mm}^2$

$$A_c = A_g - A_{sc} = 159043 - A_{sc} = 159.04 \times 10^3 - A_{sc}$$

$$1800 \times 10^3 = 0.4 \times 25\left(159.04 \times 10^3 - A_{sc}\right) + 0.67 \times 415 A_{sc}$$

$$1800 \times 10^3 = 1590.4 \times 10^3 - 10 A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = \frac{209.6 \times 10^3}{268.05} = 781.94 mm^2$$

$$\% \text{ of steel, } p_t = \frac{100 A_{sc}}{A_g} = \frac{100 \times 781.94}{159043} = 0.49\%$$

Percentage of steel is less than 0.8%.

∴ Providing minimum longitudinal reinforcement of 0.8%. ($A_{sc} = 0.8\%$ of gross cross sectional area)

$$\therefore A_{sc} = \frac{0.8}{100} \times A_g = 0.008 \times 159043 = 1272.34\text{mm}^2$$

Providing 16 mm dia bars

$$\text{No. of bars} = \frac{1272.34}{\dfrac{\pi (16)^2}{4}} = 6.34 \text{ say } 8$$

Provide 8 bars of 16 mm diameter as longitudinal reinforcement

Step:5

Design of Lateral ties:

- The diameter of lateral ties should be greater of the following:

a) 6mm

b) $\frac{1}{4} \times 16 = 4$ mm

Providing 6mm dia lateral ties.

- Spacing of lateral ties should be least of the following

a) Least lateral dimension $= D = 450$ mm

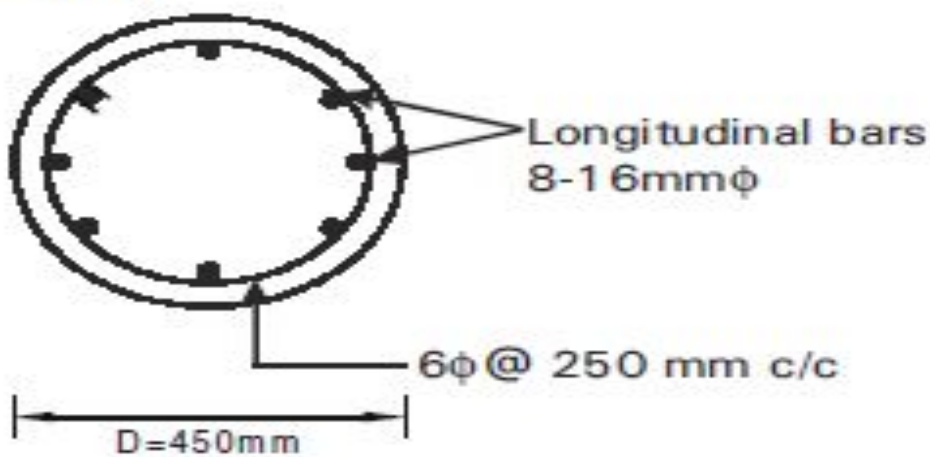
b) $16 \times$ dia of main bar $= 16 \times 16 = 256$ mm

c) 300 mm

$\therefore$ Provide 6 mm dia at 250 mm c/c

Step:6

Reinforcement details (C/S)



7. Design an RC Rectangular short column to resist an axial load of 800 kN. Use M-20 concrete and Fe-415 steel.

Solution:

Given: Axial load, $P = 800$ kN, $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step:1

Factored axial load, $P_u = 1.5 \times P = 1.5 \times 800 = 1200\text{kN}$

Step:2

Area of steel

Assuming 0.8% of gross cross sectional area

$$A_{sc} = 0.8\% \times A_g = \frac{0.8}{100} \times A_g = 0.008A_g$$

Step:3

Area of Concrete

$$A_c = A_g - A_{sc} = A_g\ 0.008A_g = A_g\ (1 - 0.008) = 0.992A_g$$

Step:4

Determine the size of column

w.k.t. $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$$1200 \times 10^3 = 0.4 \times 20 \times 0.992\ A_g + 0.67 \times 415 \times 0.008A_g$$

$$A_g = \frac{1200 \times 10^3}{10.16} = 118110\text{mm}^2$$

w.k.t. $A_g = b \times D$

Assume $b = 300$mm

$$\therefore\ D = \frac{A_g}{b} \times \frac{118110}{300} = 393.7\text{mm say } 400\text{mm}$$

$\therefore$ Provide 300 × 400 mm size column section.

Step:5

Design of Longitudinal reinforcement

$$A_{sc} = 0.008 \times A_g = 0.008 \times 300 \times 400$$
$$= 960\text{mm}^2$$

Providing 20 mm dia bars

$$\text{No. of bars} = \frac{960}{\frac{\pi(20)^2}{4}} = 3.05 \text{ say } 4$$

∴ Provide 4 bars of 20 mm dia as longitudinal reinforcement.

Step:6

Design of Lateral ties:

➢ Diameter should be greater of the following two:

i) 6mm

ii) $\frac{1}{4}$ x 20 = 5 mm Say 6 mm

➢ Spacing of lateral ties: Least of the following:

i) Least lateral dimension $b = 300\ mm$

ii) $16 \times$ dia of main bar $= 16 \times 20 = 320\ mm$

iii) $300\ mm$

∴ Provide 6 mm dia at 300 mm c/c.

8. Design an R.C.C short square column to the following particulars.

1) **Axial load** = 1200 kN.
2) **Grade of concrete** = M – 20.
3) **Length of column** = 1.85m
4) **Grade of steel** = Fe – 415.

Sketch the reinforcement details

Solution:

Given : Axial load = 1200 kN, $f_{ck} = 20\text{N/mm}^2$, $f_y = 415\text{N/mm}^2$ and $l = 1.85\text{m}$

Step:1

Factored axial load, $P_u = 1.5 \times 1200 = 1800\ kN$

Step:2

Area of Steel

Assuming 0.8% steel of column area

$$A_{sc} = 0.8\%\ A_g = 0.008A_g$$

Step:3

Area of Concrete

$$A_c = A_g - A_{sc} = 0.992A_g$$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1800 \times 10^3 = 0.4 \times 20 \times 0.992\ A_g + 0.67 \times 415 \times 0.008A_g$$

$$A_g = \frac{1800 \times 10^3}{10.16} = 177.16 \times 10^3\text{mm}^2$$

For square column, Size of column, $\sqrt{A_g} = \sqrt{177.16\times10^3} = 420.90mm$ say 450mm

$\therefore$ Provide a column of size 450 × 450 mm (i.e., $b = D = 450$mm)

Step:5

Check for slenderness ratio

Assuming both ends hinged, $l = l_{eff}$

$$\frac{l_{eff}}{D} = \frac{1850}{450} = 4.11 < 12$$

$\therefore$ The column is a short column

Step:6

Check for minimum eccentricity (IS:456-2000, Clause 25.4)

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{1850}{500} + \frac{450}{30} = 18.70mm$$

$\therefore$ Consider Minimum eccentricity as 20mm

$$e_{permissible} \leq 0.05D = 0.05 \times 450 = 22.5\text{mm}$$

$\therefore$ $e_{min} < e_{per}$, Design is safe (The column can be designed as axially loaded short column)

Step:7

Design of Longitudinal reinforcement

$$A_{sc} = 0.008\times450\times450 = 1620mm^2 \qquad |A_g = b\times D$$

Providing 20 mm dia bars

$$\text{No. of bars} = \frac{1620}{\frac{\pi(20)^2}{4}} = 5.15 \text{ say } 6$$

∴ Provide 6 bars of 20 mm diameter as longitudinal reinforcement.

Step:8

Design of lateral ties

- Diameter of Lateral ties: Greater of the following

i) 6mm

ii) $\frac{1}{4} \times 20 = 5\ mm$ Say $6\ mm$

- Spacing of lateral ties: Least of the following

i) Least lateral dimension $b = 450\ mm$

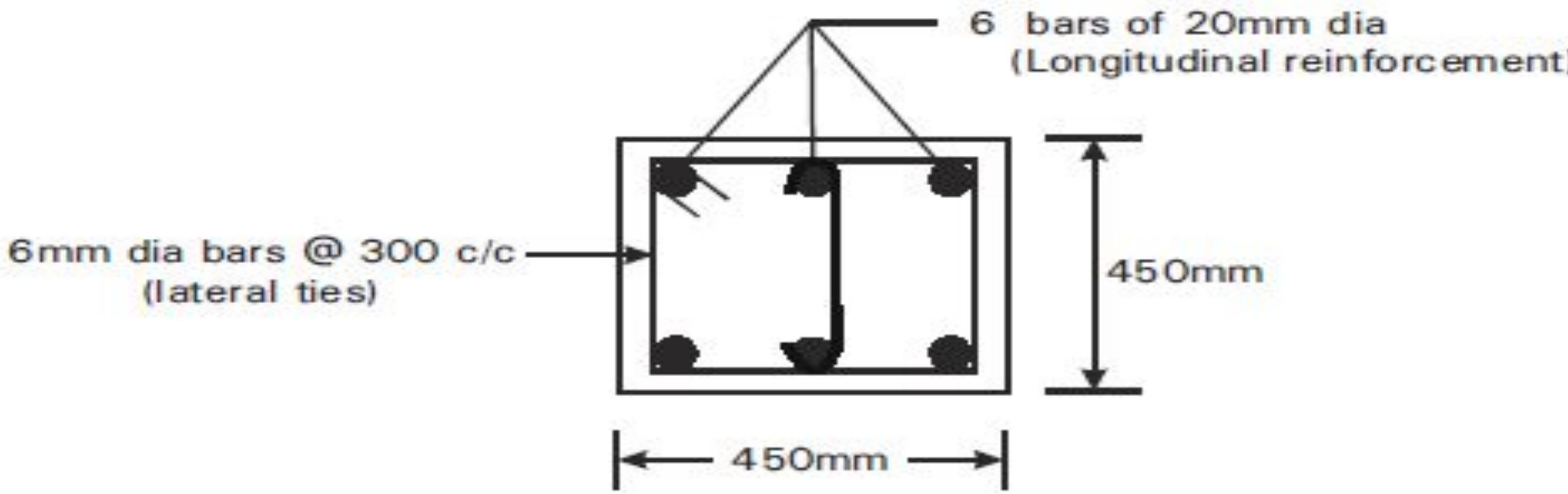
ii) $16 \times$ dia of main bar $= 16 \times 20 = 320\ mm$

iii) $300\ mm$

∴ Provide 6 *mm* dia at 300 *mm* c/c.

Step:9

Reinforcement Details (C/S)



9. Design a short R.C.C. column to carry an axial load of 1600kN. It is 4m long, effectively held in position and restrained against rotation at both ends. Use M20 concrete and Fe 415 steel.

Solution:

Given : Axial load =1600 kN, $l = 4\text{m}, f_{ck} = 20\text{N/mm}^2,\ f_y = 415\text{N/mm}^2$

Step:1

Factored axial load, $P_u = 1.5 \times 1600 = 2400$ kN

Step:2

Area of steel

Assuming 1% steel of column area,

$$A_{sc} = 1\%\ A_g = 0.01A_g$$

Step:3

Area of Concrete

$$A_c = A_g - A_{sc} = A_g - 0.01A_g = A_g\ (1-0.01) = 0.99A_g$$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2400 \times 10^3 = 0.4 \times 20 \times 0.99A_g + 0.67 \times 415 \times 0.01A_g$$

$$A_g = \frac{2400 \times 10^3}{10.70} = 224.30 \times 10^3 \text{mm}^2$$

$\Rightarrow$ For Square column, size of column = $\sqrt{A_g}$

$= \sqrt{224.30 \times 10^3} = 473.60mm$ say 500mm

$\therefore$ Provide a column of size 500 × 500 mm (i.e. $b = D = 500$mm)

Step:5

Check for slenderness ratio

For effectively held in position and restrained against rotation at both ends,

Effective length, $l_{eff} = 0.65l$ (As per IS 456:2000, Table 28)

$$l = 4\text{m} = 4000\text{mm}$$

$$\therefore l_{eff} = 0.65 \times 4000 = 2600mm$$

$$\frac{l_{eff}}{D} = \frac{2600}{500} = 5.2 < 12$$

Therefore, the column is a short column.

Step:6

Check for minimum eccentricity

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{4000}{500} + \frac{500}{30} = 24.67mm$$

$$e_{per} = 0.05D = 0.05 \times 500 = 25mm$$

$e_{min} < e_{permissible}$, design is safe

Therefore, the column can be designed as axially loaded short column.

Step:7

Design of Longitudinal reinforcement

$$A_{sc} = 0.01A_g = 0.01 \times 500 \times 500 = 2500\text{mm}^2 \qquad | A_g = b \times D$$

Providing 20 mm dia bars

$$\text{No. of bars} = \frac{2500}{\frac{\pi(20)^2}{4}} = 7.95 \text{ say } 8$$

∴ Provide 8 bars of 20 mm diameter as longitudinal reinforcement

Step:8

Design of Lateral ties

- The diameter of lateral ties should be greater of the following:

a) 6mm

b) $\frac{1}{4} \times 20 = 5$ mm

Providing 6mm dia lateral ties.

- Spacing of lateral ties

Least of the following:

a) Least lateral dimension $b = 500$ mm

b) $16 \times$ dia of main bar $= 16 \times 20 = 320$ mm

c) 300 mm

∴ Provide 6 mm dia at 300 mm c/c

10. Design the short column, axially loaded rectangular column to support a load of 875 kN. One side of the column is restricted to 300mm. Use M25 concrete and Fe415 steel.

Solution:

Given : P = 875kN, Size of the cloumn on one side = 300mm, f_{ck} = 25N/mm^2 and $f_y = 415\text{N/mm}^2$

Step:1

Factored load, $P_u = 1.5 \times 875 = 1312.5\text{kN}$

Step:2

Area of Steel

Let the gross area of the column is A_g

Assuming 1% of steel,

$$A_{sc} = 1\%\, A_g = 0.01A_g$$

Step:3

Area of concrete, $A_c = A_g - A_{sc} = A_g - 0.01A_g = 0.99A_g$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1312.5 \times 10^3 = 00.4 \times 25 \times 0.99A_g + 0.67 \times 415 \times 0.01A_g$$

$$11312.5 \times 10^3 = 12.68A_g$$

$$A_g = 103509.5\text{mm}^2$$

Since the column is rectangular and one side of the column is 300 mm, dimension of the other side $= \dfrac{A_g}{300} = \dfrac{103509.5}{300} = 345\text{mm}$, say 350mm

$\therefore$ Adopt 300mm × 350mm rectangle column.

Step:5

Design of Longitudinal reinforcement

$$A_{sc} = 0.01 \times A_g = 0.01 \times 300 \times 350 = 1050\text{mm}^2$$

Providing 16 mm dia bars

$$\therefore \text{No. of bars} = \frac{1050}{\frac{\pi(16)^2}{4}} = 5.22 \text{ say } 6$$

$\therefore$ Provide 6 bars of 16mm diameter as longitudinal reinforcement

Step:6

Design of Lateral ties :

- diameter of lateral ties should not be less than

i) $\frac{\phi_L}{4} = \frac{1}{4} \times 16 = 4mm$

ii) 6mm

Hence, adopt 6mm diameter bars

- Pitch of the ties shall be minimum of

i) least lateral dimension of column $b = 300$ mm

ii) 16 times the dia of longitudinal bar $= 16 \times 16 = 320mm$

iii) 300 mm

$\therefore$ Provide 6 mm lateral ties at 300 mm c/c

11. Design a circular short column to carry a axial load of 2500kN. Use M25 concrete and Fe 415 steel.

Solution:

Given : Axial load = 2500 kN, f_{ck} = 25N/mm² and f_y = 415N/mm²

Step:1

Factored axial load, $P_u = 1.5 \times 2500 = 3750$kN

Step:2

Area of Steel

Assuming 1% steel of column area,

$A_{sc} = 1\%\ A_g = 0.01A_g$

Step:3

Area of Concrete, $A_c = A_g - A_{sc} = A_g - 0.01A_g = 0.99A_g$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3750 \times 10^3 = 0.4 \times 25 \times 0.99A_g + 0.67 \times 415 \times 0.01A_g$$

$$A_g = \frac{3750 \times 10^3}{12.68} = 295.74 \times 10^3 \text{mm}^2$$

Area of circular column, $A_g = \frac{\pi D^2}{4}$

Diameter of column, $D = \sqrt{\frac{4A_g}{\pi}} = \sqrt{\frac{4 \times 295.74 \times 10^3}{\pi}} = 613.63$ say 650mm

Step:5

Design of Longitudinal reinforcement

$$A_{sc} = 0.01 \times A_g = 0.01 \times \frac{\pi \times 650^2}{4} = 3318.30\text{mm}^2$$

Providing 25 mm dia bars

$$\text{No of bars} = \frac{3318.30}{\frac{\pi(25)^2}{4}} = 6.76 \text{ say } 8$$

$\therefore$ Provide 8 bars of 25 mm diameter as longitudinal reinforcement

Step:6

Design of Lateral ties

- The diameter of lateral ties should be greater of the following:

i) 6mm

ii) $\frac{1}{4}$ x 25 =6.25 mm

Providing 8mm dia lateral ties.

- Spacing of lateral ties

Least of the following:

i) Least lateral dimension, $D = 650$ mm

ii) $16 \times$ dia of main bar $= 16 \times 25 = 400$ mm

iii) 300 mm

$\therefore$ Provide 8 mm dia at 300 mm c/c

12. Design a circular column to carry an axial load 1000kN using lateral ties. Use M20 grade concrete and Fe415 steel.

Solution:

Given : $P = 1000\text{kN}$, $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step:1

Factored load, $P_u = 1.5 \times 1000 = 1500\text{kN}$

Step:2

Area of Steel

Assuming 1% of steel,

$$A_{sc} = 1\% \, A_g = 0.01A_g$$

Step:3

Area of Concrete, $A_c = A_g - A_{sc} = A_g - 0.01A_g = 0.99A_g$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1500 \times 10^3 = 0.4 \times 20 \times 0.99A_g + 0.67 \times 415 \times 0.01A_g$$

$$1500 \times 10^3 = 7.92A_g + 2.78A_g = 10.7A_g$$

$$A_g = 140186.9\text{mm}^2$$

$$\text{Area of circular column, } A_g = \frac{\pi D^2}{4}$$

$$\text{Diameter of the circular column, } D = \sqrt{\frac{4A_g}{\pi}} = \sqrt{\frac{4 \times 140186.9}{\pi}} = 422.5\text{mm}$$

Adopt circular column of diameter 450mm

Step:5

Design of Longitudinal reinforcement

$$A_{sc} = 0.01A_g,\ A_{sc} = \frac{0.01 \times \pi \times 450^2}{4} = 1590.43mm^2$$

Providing 16 mm dia bars

$$\text{No. of bars} = \frac{1590.43}{\frac{\pi(16)^2}{4}} = 7.9 \text{ say } 8$$

Step:6

Design of Lateral ties

- Diameter of lateral ties should not be less than

i) $\frac{\phi_L}{4} = \frac{1}{4} \times 16 = 4mm$

ii) 6mm

Hence, adopt 6mm diameter bars

- Pitch of the ties shall be minimum of

i) least lateral dimension of column $D = 450$ mm

ii) 16 times the dia of longitudinal bar = $16 \times 16 = 256mm$

iii) 300 mm

∴ Provide 6 mm lateral ties at 250 mm c/c.

13. **Calculate load carrying capacity of short column 300 mm × 450 mm in size reinforced with 4-16 mm and 4-12 mm diameter bars. Use M20 and Fe 415 steel.**

Solution:

Given: $b = 300\text{mm}, D = 450\text{mm}, A_{sc} = 4 - 16\phi + 4 - 12\phi, f_{ck} = 20\text{N/mm}^2, f_y = 415\text{N/mm}^2$

Step:1

Area of reinforcement provided, $A_{sc} = 4 \times \frac{\pi \times 16^2}{4} + 4 \times \frac{\pi \times 12^2}{4} = 1256.63 mm^2$

Step:2

Load carrying capacity for axially loaded short column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$A_c = A_g - A_{sc} = b \times D - A_{sc} = 300 \times 450 - 1256.63 = 133743.37\text{mm}^2$

$\therefore P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} = 0.4 \times 20 \times 133743.37 + 0.67 \times 415 \times 1256.63$

$\therefore P_u = 1419352\text{N} = 1419.35\text{kN}$

14. Design a circular column to carry an axial load of 1500 kN using lateral ties. The unsupported length of column = 3.3m. Check for slenderness of column and minimum eccentricity. Sketch the reinforcement details.

Solution

Given: $P = 1500$ kN, $l = 3.3$m

Step:1

Factored axial load, $P_u = 1.5 \times P = 1.5 \times 1500 = 2250$kN

Step:2

Area of Steel

Assuming 1% steel of column area

$\therefore A_{sc} = 1\% \, A_g = 0.01 A_g$

Step:3

Area of concrete

$A_c = A_g - A_{sc} = A_g - 0.01\, A_g = 0.99 A_g$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Assume $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415 \text{ N/mm}^2$

$$2250 \times 10^3 = 0.4 \times 20 \times 0.99\, A_g + 0.67 \times 415 \times 0.01\, A_g$$

$$2250 \times 10^3 = 10.70\, A_g$$

$$\therefore A_g = \frac{2250 \times 10^3}{10.70} = 210.28 \times 10^3 \text{mm}^2$$

Area of circular column, $A_g = \dfrac{\pi D^2}{4}$

$\therefore$ Diameter of the circular column, $D = \sqrt{\dfrac{4A_g}{\pi}} = \sqrt{\dfrac{4 \times 210.28 \times 10^3}{\pi}}$

$$\therefore \quad D = 517.43\text{mm},$$

Adopt circular column of diameter 550mm

Step:5

Check for slenderness ratio

Unsupported length of column, $l = 3.3\text{m}$

Assuming both ends hinged, $l = l_{eff} = 3300\text{mm}$

$$\frac{l_{eff}}{D} = \frac{3300}{550} = 6 < 12$$

$\therefore$ The column is a short column

Step:6

Check for minimum eccentricity

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$e_{min} = \frac{3300}{500} + \frac{550}{30} = 24.93\text{mm}$$

$$e_{permissible} \leq 0.05\,D = 0.05 \times 550 = 27.5\text{mm}$$

$\therefore e_{min} < e_{permissible}$, Design is safe

$\therefore$ The column can be designed as axially loaded short column

Step:7

Design of Longitudinal Reinforcement

$$A_{sc} = 0.01\,A_g = 0.01 \times \frac{\pi \times 550^2}{4} = 2375.83\ \text{mm}^2$$

Providing 20mm dia bars

$$\text{No. of bars} = \frac{2375.83}{\frac{\pi(20)^2}{4}} = 7.56\text{, say } 8$$

$\therefore$ Provide 8 bars of 20 mm diameter as longitudinal reinforcement

Step:8

Design of Lateral ties

- Diameter of lateral ties should be greater of the following

(i) $\frac{1}{4} \times 20 = 5\text{mm}$

(ii) 6mm

Providing 6mm dia lateral ties

- Spacing of lateral ties should be least of the following

(i) Least lateral dimension = D = 550mm

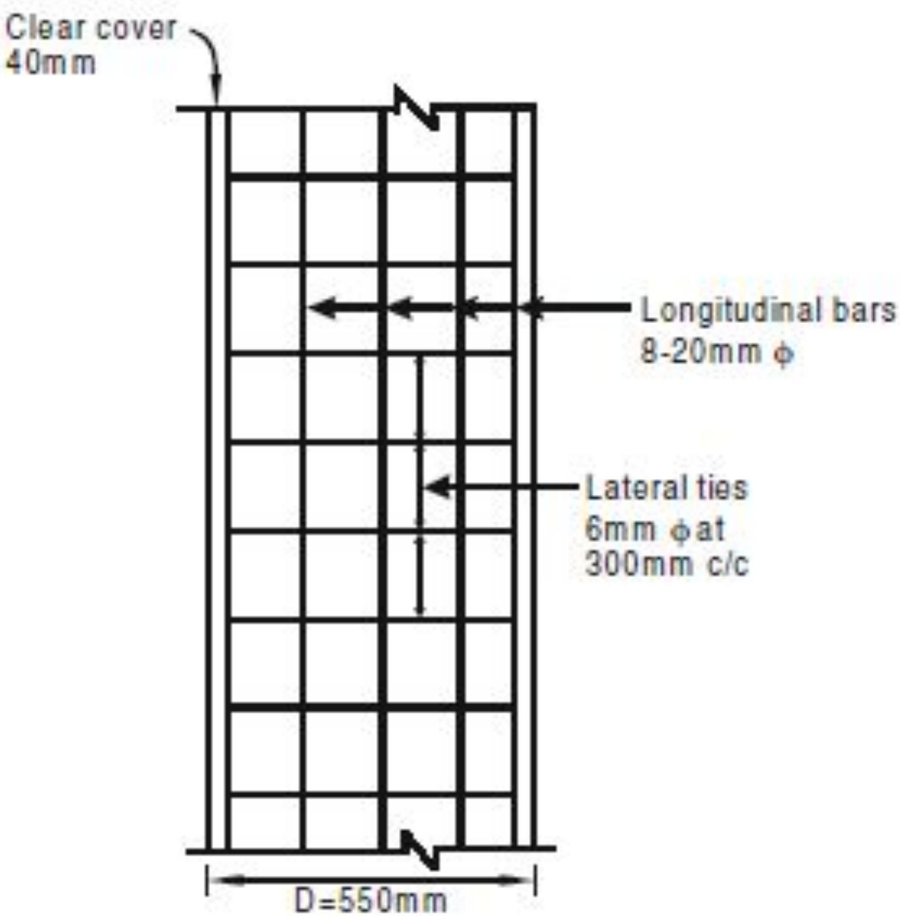
(ii) $16 \times$ dia of main bar = $16 \times 20 = 320$mm

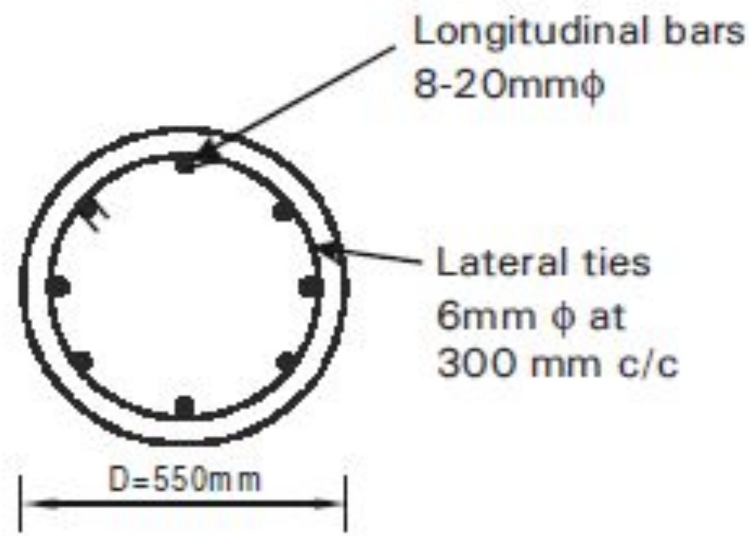
(iii) 300mm

$\therefore$ Provide 6 mm dia at 300 mm c/c

Step:9

Reinforcement details (*L/S* and *C/S*)





15. Design RCC column having unsupported length 2.75m to support a load of 2000kN, using M20 concrete and *Fe*415 steel (i) As a square section, (ii) As a rectangular section with $\frac{b}{D} = \frac{3}{4}$.

Solution

Given: $P = 2000$ kN, $l = 2.75$m, $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step:1

Factored axial load, $P_u = 1.5 \times 2000 = 3000$kN

Step:2

Area of Steel

Assuming 0.8% of gross cross sectional area

$$\therefore A_{sc} = 0.8\% \, A_g = \frac{0.8}{100} \times A_g = 0.008 A_g$$

Step:3

Area of concrete

$$A_c = A_g - A_{sc} = A_g - 0.008 \, A_g = 0.992 A_g$$

Step:4

Determine the size of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3000 \times 10^3 = 0.4 \times 20 \times 0.992\, A_g + 0.67 \times 415 \times 0.008 A_g$$

$$3000 \times 10^3 = 10.16 A_g$$

$$A_g = \frac{3000 \times 10^3}{10.16} = 295.26 \times 10^3 \text{mm}^2$$

Case (i)

As a square section ($A_g = b \times D$)

- **Size of column,** $b = D = \sqrt{A_g} = \sqrt{295.26 \times 10^3} = 543.38$mm say 550mm

  $\therefore$ Provide a column of size 550 × 550mm (i.e., $b = D = 550$mm)

- **Check for slenderness ratio**

  Assuming both ends hinged, $l = l_{eff} = 2.75$m

  $$\frac{l_{eff}}{D} = \frac{2750}{550} = 5 < 12$$

  $\therefore$ The column is a short column

- **Check for minimum eccentricity**

  $$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

  $$= \frac{2750}{500} + \frac{550}{30} = 23.83\text{mm}$$

  $e_{permissible} \leq 0.05\, D = 0.05 \times 550 = 27.5$mm

  $\therefore e_{min} < e_{permissible}$, Design is safe

  $\therefore$ The column can be designed as axially loaded short column

- **Design of Longitudinal reinforcement**

  $A_{sc} = 0.008A_g = 0.008 \times b \times D = 0.008 \times 550 \times 550 = 2420\text{mm}^2$

  Providing 20mm dia bars

  $$\text{No of bars} = \frac{2420}{\frac{\pi}{4}(20)^2} = 7.70 \text{ say } 8$$

  $\therefore$ Provide 8 bars of 20mm diameter as longitudinal reinforcement.

- **Design of lateral ties**

  ➢ Diameter of lateral ties should be greater of the following

  (i) $\frac{1}{4} \times 20 = 5\text{mm}$

  (ii) 6mm

  Say 6mm

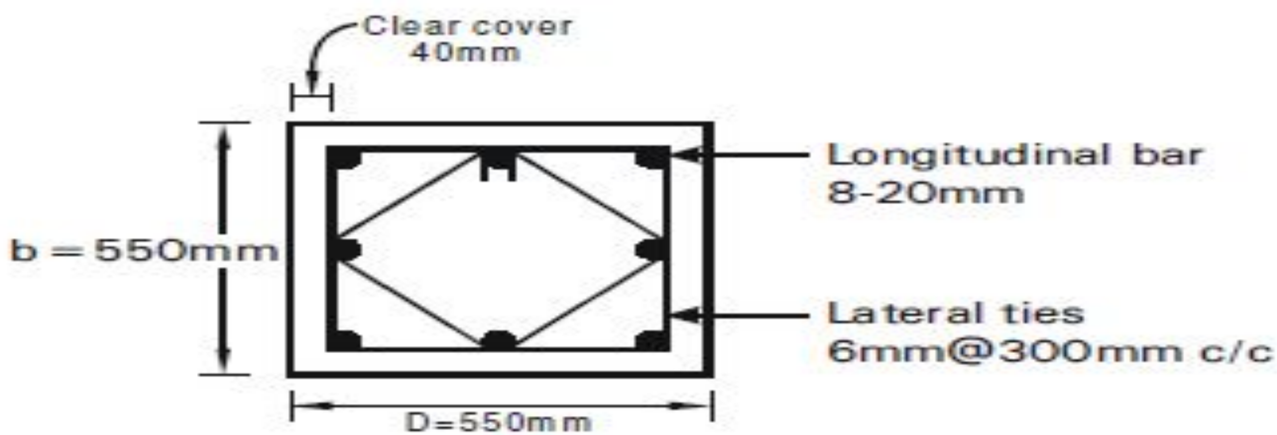
  ➢ Spacing of lateral ties should be least of the following

  (i) least lateral dimension = $b = D = 550$mm

  (ii) 16 × dia of main bar = 16 × 20 = 320 mm

  (iii) 300mm

  $\therefore$ Provide 6mm dia at 300 mm c/c



Case (ii)

As a rectangular section with $\frac{b}{D} = \frac{3}{4}$

Here $A_g = 295.26 \times 10^3 mm^2$

W.K.T $A_g = b \times D$

Given $\frac{b}{d} = \frac{3}{4}$

$$\Rightarrow b = \frac{3}{4} \times D = 0.75\ D$$

Similarly, $b = \frac{A_g}{D}$

$$\Rightarrow 0.75\ D \times D = A_g$$

$$0.75\ D^2 = A_g$$

$$\therefore D = \sqrt{\frac{A_g}{0.75}} = \sqrt{\frac{295.26 \times 10^3}{0.75}} = 627.4 mm \text{ say } 650 mm$$

$\therefore$ Now, $b = 0.75 \times D = 0.75 \times 650 = 487.5 mm$ say $500 mm$

$\therefore$ Provide a column of size $500 \times 650 mm$ (i.e., $b \times D$)

- **Check for slenderness ratio**

  Assuming both ends hinged, $l = l_{eff} = 2.75\text{m}$

  $$\frac{l_{eff}}{b} = \frac{2750}{500} = 5.5 < 12$$

  $$\frac{l_{eff}}{D} = \frac{2750}{650} = 4.23 < 12$$

  $\therefore$ The column is a short column

- **Check for minimum eccentricity**

  ➢ Along longer direction

  $$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

  $$= \frac{2750}{500} + \frac{650}{30} = 27.16\text{mm}$$

  $e_{permissible} \leq 0.05\ D = 0.05 \times 650 = 32.5\text{mm}$

  $\Rightarrow e_{min} < e_{permissible}$, Design is safe

  ➢ Along shorter direction

  $$e_{min} = \frac{l}{500} + \frac{b}{30} \geq 20\text{mm}$$

  $$= \frac{2750}{500} + \frac{500}{30} = 22.16\text{mm}$$

  $e_{permissible} \leq 0.05\ b = 0.05 \times 500 = 25\text{mm}$

  $\Rightarrow e_{min} < e_{permissible}$, Design is safe

  $\therefore$ The column can be designed as axially loaded short column

- **Design of Longitudinal reinforcement**

$A_{sc} = 0.008\, A_g = 0.008 \times b \times D = 0.008 \times 500 \times 650 = 2600\text{mm}^2$

Providing 25mm dia bars

$$\text{No. of bars} = \frac{2600}{\frac{\pi}{4}(25)^2} = 5.29 \text{ say } 6$$

$\therefore$ Provide 6 bars of 25mm diameter as longitudinal reinforcement.

- **Design of lateral ties**

➢ Diameter of lateral ties should be greater of the following

(i) $\frac{1}{4} \times 25 = 6.25\text{mm}$

(i) 6 mm, Say 8mm

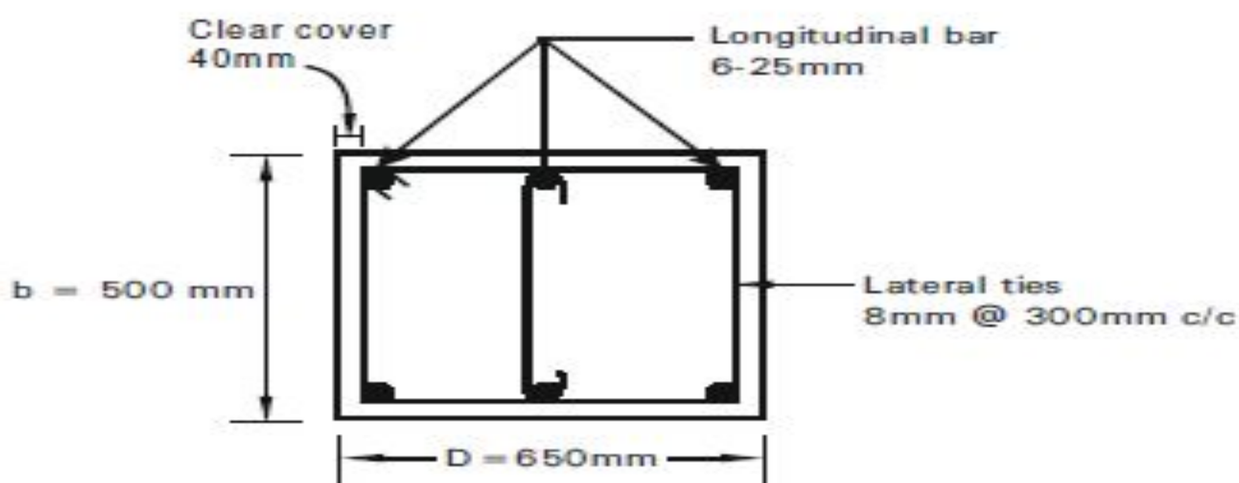
➢ Spacing of lateral ties should be least of the following

(i) least lateral dimension = b = 500mm

(ii) 16 × dia of main bar = 16 × 25 = 400mm

(iii) 300mm

$\therefore$ Provide 8mm dia at 300mm c/c



16. Design a square and circular column to carry a working load of 1000kN. The grade of concrete and steel are M20 and Fe415 respectively. Take unsupported length as 3.5m.

Solution

Gievn: $P = 1000$ kN, $f_{ck} = 20\text{N/mm}^2$, $f_y = 415\text{N/mm}^2$ and $l = 4$m

Step:1

Factored axial load, $P_u = 1.5 \times P = 1.5 \times 1000 = 1500$kN

Step:2

Area of Steel

Assuming 1% steel of column area

$\therefore A_{sc} = 1\% \, A_g = 0.01A_g$

Step:3

Area of concrete

$A_c = A_g - A_{sc} = A_g - 0.01 \, A_g = 0.99A_g$

Step:4

Determine the size of column

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc}$$

$$1500 \times 10^3 = 0.4 \times 20 \times 0.99 \, A_g + 0.67 \times 415 \times 0.01A_g$$

$$1500 \times 10^3 = 10.7A_g$$

$$\therefore A_g = \frac{1500 \times 10^3}{10.7} = 140.18 \times 10^3\text{mm}^2$$

Case (i)

For square column

$$A_g = b \times D$$

$$\therefore\ b = D = \sqrt{A_g} = \sqrt{140.18 \times 10^3} = 374.40\text{mm}$$

$\therefore$ Provide a column of size 400 × 400mm (i.e., $b = D = 400$mm)

- **Check for slenderness ratio**

Assuming both ends hinged, $l = l_{eff} = 3.5$m

$$\frac{l_{eff}}{D} = \frac{3500}{400} = 8.75 < 12$$

$\therefore$ The column is a short column

- **Check for minimum eccentricity**

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{3500}{500} + \frac{400}{30} = 20.33\text{mm}$$

$e_{permissible} \leq 0.05\ D = 0.05 \times 400 = 20$mm

$\therefore\ e_{min} > e_{permissible}$, Design is not safe

Hence increase the size of column

$\therefore$ Let us provide size of column as 425 × 425mm

$$\Rightarrow e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20\text{mm}$$

$$= \frac{3500}{500} + \frac{425}{30} = 21.16\text{mm}$$

$e_{permissible} \leq 0.05\ D = 0.05 \times 425 = 21.25$mm

$\therefore\ e_{min} < e_{permissible}$, Design is safe

The column can be designed as axially loaded short column

- **Design of Longitudinal reinforcement**

  $A_{sc} = 0.01A_g = 0.01 \times b \times D = 0.01 \times 425 \times 425 = 1806.25\text{mm}^2$

  Providing 20mm dia bars

  $$\text{No of bars} = \frac{1806.25}{\frac{\pi}{4}(20)^2} = 5.75 \text{ say } 6$$

  $\therefore$ Provide 6 bars of 20mm diameter as longitudinal reinforcement.

- **Design of lateral ties**

  ➢ Diameter of lateral ties should be greater of the following

  (i) $\frac{1}{4} \times 20 = 5\text{mm}$

  (ii) 6mm, say 6mm

  ➢ Spacing of lateral ties should be least of the following

  (i) least lateral dimension $= b = D = 425$mm

  (ii) $16 \times$ dia of main bar $= 16 \times 20 = 320$ mm

  (iii) 300mm

  $\therefore$ Provide 6mm dia at 300 mm c/c

Case (ii)

For Circular column

$$A_g = 140.18 \times 10^3 mm^2$$

$$\therefore A_g = \frac{\pi}{4} D^2$$

$\therefore$ Diameter of the circular column, $D = \sqrt{\frac{4A_g}{\pi}} = \sqrt{\frac{4 \times 140.18 \times 10^3}{\pi}}$

$= 422.47mm$

$\therefore$ Adopt circular column of diameter 425mm

- **Design of Longitudinal reinforcement**

$A_{sc} = 0.01\ A_g = 0.01 \times \frac{\pi}{4} \times 425^2 = 1418.62mm^2$

Providing 16mm dia bars

No. of bars $= \frac{1418.62}{\frac{\pi}{4}(16)^2} = 7.05$ say 8

$\therefore$ Provide 8 bars of 16mm diameter as longitudinal reinforcement

- **Design of lateral ties**
  - ➢ Diameter of lateral ties should be greater of the following

(i) $\frac{1}{4} \times 16 = 4mm$

(ii) 6mm, say 6mm

➢ Spacing of lateral ties should be least of the following

(i) least lateral dimension $= D = 425mm$

(ii) $16 \times$ dia of main bar $= 16 \times 16 = 256$ mm

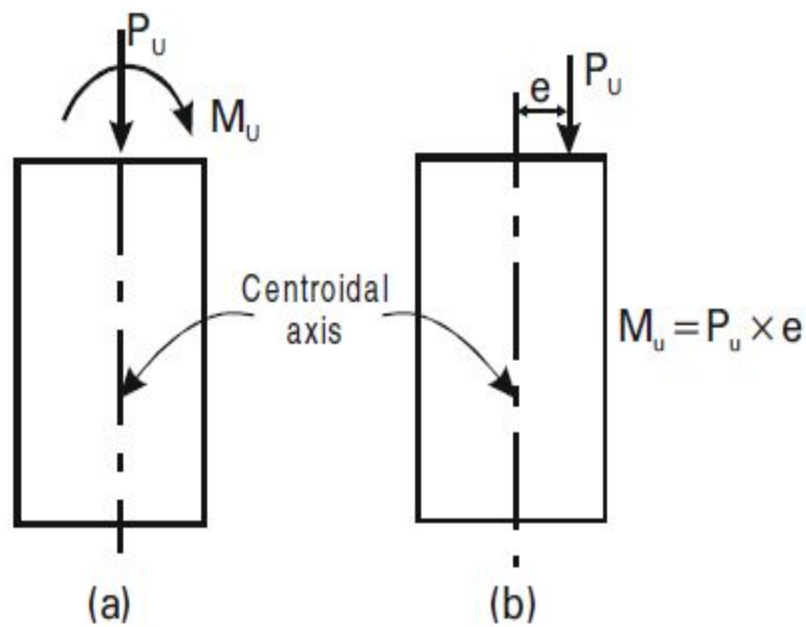
(iii) 300mm

$\therefore$ Provide 6mm dia at 250 mm c/c

CONCEPT OF BI-AXIAL BENDING

The load on the column is rarely axial. There is always some minimum inherent eccentricity on account of inaccuracies in loading, bad workmanship etc.

In such a cases the column is subjected to axial compression P_u and bending moment M_u. This loading system can be reduced to a single resultant P_u acting at an eccentricity $e = \frac{M_u}{P_u}$.



But the design of member subjected to combined axial load and uniaxial bending involves lengthy calculation by trial and error. In order to overcome these difficulties, interaction diagrams may be used. These have been prepared and published by BIS in "SP:16 Design aids for Reinforced Concrete to IS: 456-2000."

STEPS FOR DESIGN OF COLUMN SUBJECTED TO COMBINED AXIAL COMPRESSION AND UNIAXIAL BENDING

For determination of the area of steel required for the predetermined section to support the specified P_u and M_u following procedure can be adopted:

Pre-determine the trial cross-sectional dimensions, distribution of reinforcement and its effective cover. Check whether the column is short or long. If short proceed as follows :

Steps

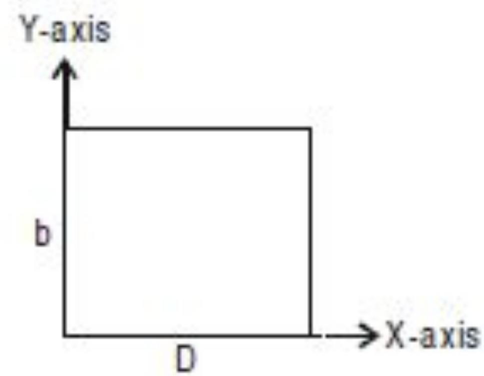
i) Check the eccentricity:

$$e = \frac{M_u}{P_u}$$

$$e_{min} = \frac{l}{500} + \frac{D}{30} \text{ or } 20\text{mm}$$

if $e > e_{min}$

Then column is designed as short column subjected to axial load and bending otherwise column is designed as axially loaded column.



ii) Compute the design parameters:

$$\frac{d'}{D}, \frac{P_u}{f_{ck}bD} \text{ and } \frac{M_u}{f_{ck}bD^2}$$

iii) For the given $\frac{d'}{D}$ ratio, grade of steel, shape of the section (rectangular and circular) and the type of distribution of steel (2 sided and 4 sided reinforcement), choose the appropriate curve.

iv) For the computed values of $\frac{P_u}{f_{ck}bD}$ and $\frac{M_u}{f_{ck}bD^2}$ mark the points on the curve selected in step (iii) and find the value of $\frac{p}{f_{ck}}$.

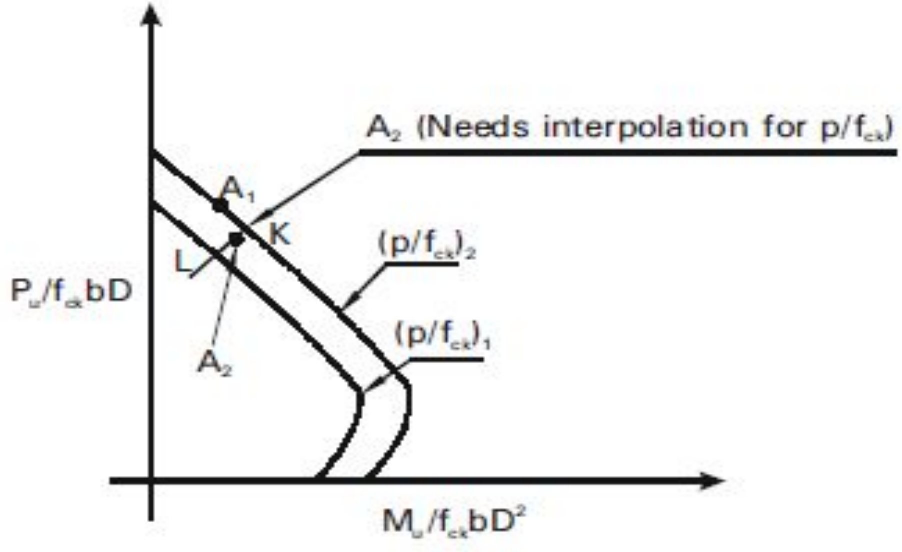


Fig. 5.11 : Determination of $\frac{p}{f_{ck}}$ by linear interpolation (not to scale)

(Note: The two parameters $\frac{P_u}{f_{ck}bD}$ and $\frac{M_u}{f_{ck}bD^2}$ are known and the point A is located on the design chart with these two coordinates (Fig. 5.11). The point may be like A_1, on a particular curve of specified $\frac{p}{f_{ck}}$, or like A_2, in between two such curves having two values of $\frac{p}{f_{ck}}$, the difference between the two values of $\frac{p}{f_{ck}}$ is 0.02. In the first case, the corresponding $\frac{p}{f_{ck}}$ is obtained directly as specified on the curve. While, in the second case, liner interpolation is to be done by drawing a line KL perpendicular to the two curves and passing through the point A_2.)

v) Calculate the total area of reinforcement for the value of $\frac{p}{f_{ck}}$ obtained in step (iv) as follows:

Percentage steel, $p =$ Value of $\frac{p}{f_{ck}}$ obtained from curve $\times f_{ck}$

$$p = \frac{100 A_{sc}}{bD}$$

$$\therefore A_{sc} = \frac{pbD}{100}$$

Distribute total area of steel suitably according to the curve used.

vi) Design of lateral ties same as axially loaded columns.

Note :

1. External columns of multistoried buildings are subjected to direct axial loads and bending moment.
2. For bending about x-axis bisecting the depth of the column $\Rightarrow \frac{d'}{D}$ and $\frac{M_u}{f_{ck} bD^2}$
3. For bending about y-axis bisecting the width of the column the chart to be referred to is having value of d'/b and use expression $\frac{M_u}{f_{ck} b^2 D}$.

WORKED EXAMPLES ON DESIGN OF COLUMN SUBJECTED TO AXIAL COMPRESSION AND UNIAXIAL BENDING

1. **A short RCC column 300 × 400mm is reinforced with 6 bars of 20mm diameter equally distributed on the two sides. Determine the ultimate axial load which the column can take, if ultimate moment is 100kN-m and take effective cover = 40mm. Use M20 grade concrete and Fe 415 grade steel.**

   Solution:

   Given: $M_u = 100kN-m = 100\times10^6 N-mm, b = 300mm, D = 400mm, d' = 40mm$

   $$f_{ck} = 20N/mm^2, f_y = 415N/mm^2, A_{sc} = 6\times\frac{\pi\times20^2}{4} = 1885mm^2$$

Step : 1 Non dimensional parameters

$$\frac{M_u}{f_{ck}bD^2} = \frac{100\times10^6}{20\times300\times400^2} = 0.104$$

Percentage of steel $p = \frac{100A_{sc}}{bD} = \frac{100\times1885}{300\times400} = 1.57\%$

$$\frac{p}{f_{ck}} = \frac{1.57}{20} = 0.078\ ;\ \frac{d'}{D} = \frac{40}{400} = 0.10$$

Using SP:16, chart No.32

For $\frac{p}{f_{ck}} = 0.078$, $\frac{M_u}{f_{ck}bD^2} = 0.104$, and $f_y = 415$ N/mm²

Find $\frac{P_u}{f_{ck}bD}$

Step : 2 Determine the ultimate axial load (P_u)

$\frac{p}{f_{ck}}$	$\frac{P_u}{f_{ck}bD}$
0.06	0.37
0.08	0.45

By interpolation, we get, $\frac{P_u}{f_{ck}bD} = 0.37 + \frac{(0.45-0.37)}{(0.08-0.06)} \times (0.078-0.06) = 0.442$

Therefore, Factored axial load,

$P_u = f_{ck}bD \times 0.42 = 20 \times 300 \times 400 \times 0.42 = 1060800N = 1060.8kN$

2. Determine the ultimate bending moment and eccentricity at which axial load acts for a column of size 300mmx500mm reinforced with 6 bars of 20mm diameter arranged on two sides of column. It is subjected to an factored axial load of 1200kN. Use M20 concrete and Fe 415 steel. Take $d' = 50mm$.

Solution:

Given $P_u = 1200kN = 1200 \times 10^3 N, b = 300mm, D = 500mm, d' = 50mm$

$$f_{ck} = 20N/mm^2, f_y = 415N/mm^2, A_{sc} = 6 \times \frac{\pi \times 20^2}{4} = 1885mm^2$$

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{1200 \times 10^3}{20 \times 300 \times 500} = 0.40$$

Percentage of steel, $p = \frac{100A_{sc}}{bD} = \frac{100 \times 1885}{300 \times 500} = 1.25\%$

$$\frac{p}{f_{ck}} = \frac{1.25}{20} = 0.063 \;;\; \frac{d'}{D} = \frac{50}{500} = 0.10$$

Using SP:16, chart No.32

For $\frac{p}{f_{ck}} = 0.063$, $\frac{P_u}{f_{ck}bD} = 0.40$, and $f_y = 415$ N/mm²

Find $\frac{M_u}{f_{ck}bD^2}$

Step : 2 Determine the ultimate bending moment (M_u) and Eccentricity (e)

$\frac{p}{f_{ck}}$	$\frac{M_u}{f_{ck}bD^2}$
0.06	0.095
0.08	0.12

By interpolation, we get, $\frac{M_u}{f_{ck}bD^2} = 0.095 + \frac{(0.12 - 0.095)}{(0.08 - 0.06)} \times (0.063 - 0.06) = 0.099$

Therefore, Factored moment,

$$M_u = f_{ck}bD^2 \times 0.099 = 20 \times 300 \times 500^2 \times 0.099 = 148.5 \times 10^6 N - m = 148.5 kN - m$$

$$\text{Ecentricity}, e = \frac{M_u}{P_u} = \frac{148.5}{1200} = 0.1237 m = 123.7 mm$$

3. Design an reinforced concrete square column of 500mm side to carry an ultimate load of 1000kN at an eccentricity of 200mm. Use M20 concrete and Fe 415 steel.

Solution : Given: $D = b = 500mm, P_u = 1000kN = 1000\times10^3 N, e = 200mm,$

$f_{ck} = 20N/mm^2, f_y = 415N/mm^2$

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{1000\times10^3}{20\times500\times500} = 0.20$$

Factored moment, $M_u = P_u \times e = 1000\times10^3 \times 200 = 200\times10^6 N - mm$

$$\frac{M_u}{f_{ck}bD^2} = \frac{200\times10^6}{20\times500\times500^2} = 0.08$$

Assuming effective cover, $d' = 50mm$

Now, $\frac{d'}{D} = \frac{50}{500} = 0.10$

Using SP:16, chart 32 ($f_y = 415N/mm^2$ and $\frac{d'}{D} = 0.10$)

(Providing Reinforcement on two sides)

For $\frac{P_u}{f_{ck}bD} = 0.20$ and $\frac{M_u}{f_{ck}bD^2} = 0.08$, values obtain $\frac{p}{f_{ck}} = 0.02$ from chart.

Step : 2 Design of longtudinal reinfrocement

Percentage of steel, $p = 0.02 \times f_{ck} = 0.02 \times 20 = 0.4\% < 0.8\%$ (Minimum steel)

Therefore, providing 0.8% steel as longitudinal reinforcement.

$$p = \frac{100 A_{sc}}{bD}$$

$$\therefore A_{sc} = \frac{pbD}{100} = \frac{0.80 \times 500 \times 500}{100} = 2000 mm^2 \; \left(A_{sc} \text{ required}\right)$$

Providing 2-25mm diameter bars + 4-20 mm diameter bars (distributing 3 bars on either face.)

Area of reinforcement provided $A_{sc} = 2 \times \frac{\pi \times 25^2}{4} + 4 \times \frac{\pi \times 20^2}{4} = 2238.34 mm^2 > A_{sc_{required}}$

Hence (OK)

Step : 3 Design of lateral ties

Diameter and Pitch of lateral ties:

- Diameter shall not be less than

a) 1/4th diameter of the largest longitudinal bar

$$= \frac{1}{4} \times 25 = 6.25 mm \text{ say 8mm}$$

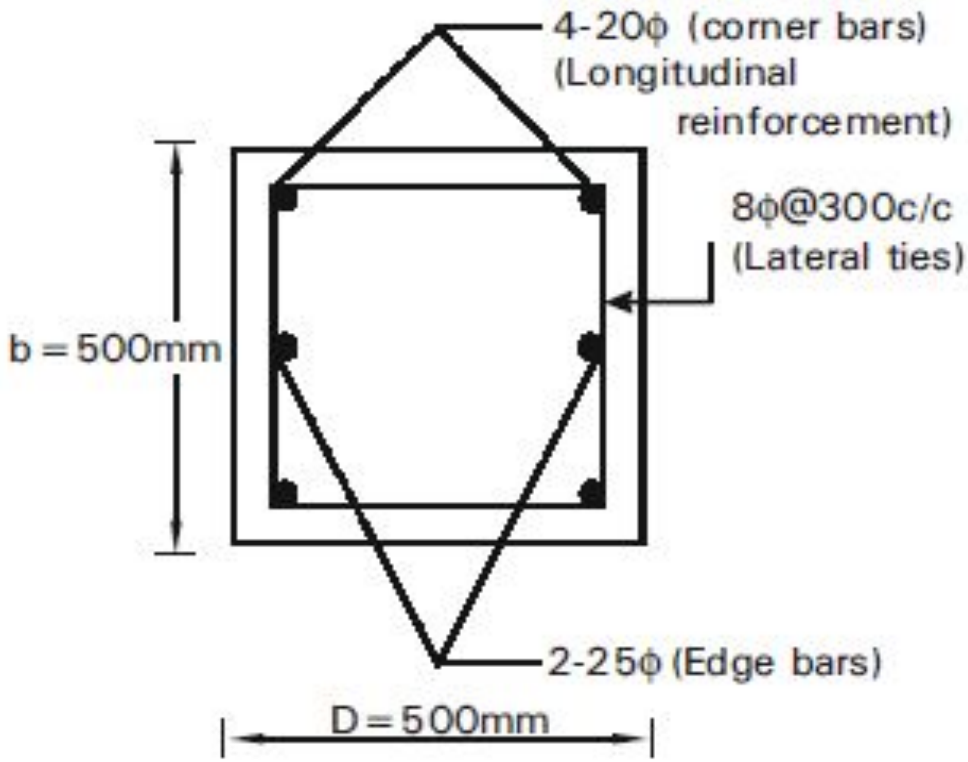
b) 6mm

Providing 8mm diameter

- The pitch of transverse reinforcement shall not more than the least of the following distances:
  a) The least lateral dimension of the column, $b = 500mm$
  b) 16 times the smallest diameter of longitudinal bar = $16 \times 20 = 320mm$
  c) 300mm

$\therefore$ Provide 8mm dia bars at 300mm c/c

Step : 4 Reinforcement details (C/S)



4. Design the reinforcement for a column of size 300mm × 400mm having effective length 2.5m. Moment about the major axis (x-axis) of the column = 100kN-m and axial load = 800kN. Use M25 concrete and Fe 500 steel. Provide the reinforcement on two sides.

Solution:

Given:

$$D = 400mm, b = 300mm, P = 800kN = 800\times10^3\,N, M = 100kN-m = 100\times10^6\,N-mm$$

$$f_{ck} = 25N/mm^2, f_y = 500N/mm^2, l_{eff} = 2.5m = 2500mm$$

Step : 1 Check for slenderness ratio

$$\frac{l_{eff}}{D} = \frac{2500}{400} = 6.25 < 12; \quad \frac{l_{eff}}{b} = \frac{2500}{300} = 8.33 < 12$$

Hence it is short column.

Step : 1 Check for eccentricity

Factored axial load, $P_u = P\times1.5 = 800\times10^3\times1.5 = 1200\times10^3\,N$

Factored moment, $M_u = M\times1.5 = 100\times10^6\times1.5 = 150\times10^6\,N-mm$

$$\text{Ecentricity}, e = \frac{M_u}{P_u} = \frac{150\times10^6}{1200\times10^3} = 125mm$$

$$\text{Minimum eccentricity, } e_{\min} = \frac{l}{500} + \frac{D}{30} = \frac{2500}{500} + \frac{400}{30} = 18.33mm \quad \geq 20\text{mm}$$

$$\therefore e > e_{\min}$$

Hence, it is designed as short column subjected to axial load and bending

Assuming effective cover $d' = 40mm$

Now, $\frac{d'}{D} = \frac{40}{400} = 0.10$

Step : 3 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{1200 \times 10^3}{25 \times 300 \times 400} = 0.40$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{150 \times 10^6}{25 \times 300 \times 400^2} = 0.125$$

Using SP:16, chart 36 ($f_y = 500 N/mm^2, \frac{d'}{D} = 0.10$)

For $\frac{P_u}{f_{ck}bD} = 0.40$ and $\frac{M_u}{f_{ck}bD^2} = 0.125$ values obtain $\frac{p}{f_{ck}} = 0.07$ from chart.

Step : 4 Design of Longitudinal reinforcement

Percentage steel, $p = 0.07 \times f_{ck} = 0.07 \times 25 = 1.75\% < 4\%$ (Maximum steel, OK)

$$p = \frac{100A_{sc}}{bD}$$

$$\therefore A_{sc} = \frac{pbD}{100} = \frac{1.75 \times 300 \times 400}{100} = 2100mm^2 \left(A_{SC} \text{ required}\right)$$

Providing 4-25mm diameter bars + 2-20mm diameter bars (distributing 3 bars on either face)

Area of reinforcement provided, $A_{sc} = 4\times\frac{\pi\times25^2}{4} + 2\times\frac{\pi\times20^2}{4} = 2591.79mm^2 > A_{sc}$ required

Step 5 : Design of lateral ties

Diameter andPitch of lateral ties:

- Diameter shall not be less than

  a) 1/4th diameter of the largest longitudinal bar = $\frac{1}{4}\times25 = 6.25mm$ say 8mm

  b) 6mm

  Providing 8mm diameter

- The pitch of transverse reinforcement shall not more than the least of the following distances:

  a) The least lateral dimension of the column, b = 300mm

  b) 16 times the smallest diameter of longitudinal bar = $16\times20 = 320mm$

  c) 300mm

  Therefore spacing = 300mm

$\therefore$ Provide 8mm dia bars at 300mm c/c

5. **Determine the reinforcement to be provided in a square column subjected to uniaxial bending with the following data :**

Size of the column = 450x450mm

Grade of concrete = M25

Yield strength of steel = 500N/mm²

Factored load = 2500kN

Factored moment = 150kN-m

Arrangement of reinforcement = On two sides.

Assume 25mm bars with 40mm cover.

Solution:

Given:

$$D = 450mm, b = 450mm, P_u = 2500kN = 2500\times10^3 N, M_u = 150kN-m = 150\times10^6 N-mm$$

$f_{ck} = 25N/mm^2, f_y = 500N/mm^2$, Nominal cover = 40mm

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{2500\times10^3}{25\times450\times450} = 0.49$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{150\times10^6}{25\times450\times450^2} = 0.065$$

Assuming 25mm ϕ longitudinal bars with 40mm nominal cover,

Effective cover, $d' = 40 + \frac{25}{2} = 52.5mm$ | $\therefore d' =$ Nominal cover $+ \frac{\phi}{2}$

Now, $\frac{d'}{D}=\frac{52.5}{450}=0.11$, consider $\frac{d'}{d}=0.10$

Using SP:16, chart 36 $\left(f_y=500N/mm^2, \frac{d'}{D}=0.10\right)$

For $\frac{P_u}{f_{ck}bD}=0.49$ and $\frac{M_u}{f_{ck}bD^2}=0.065$ values obtain $\frac{p}{f_{ck}}=0.06$ from chart.

Step : 2 Design of longitudinal reinforcement

Percentage of steel, $p=0.06\times f_{ck}=0.06\times 25=1.5\%$

$$p=\frac{100A_{sc}}{bD}$$

$$\therefore A_{sc}=\frac{pbD}{100}=\frac{1.5\times 450\times 450}{100}=3037.4mm^2$$

Providing 8 bars of 25mm ϕ equally distributed on two sides.

Area of steel provided = $\frac{8\times\pi\times 25^2}{4}$ = 3927 mm² > 3037.4 mm²

Hence (OK)

Step : 3 Design of lateral ties

Diameter and Pitch of lateral ties:

- Diameter shall not be less than

a) 1/4th diameter of the largest longitudinal bar = $\frac{1}{4} \times 25 = 6.25mm$ say 8mm

b) 6mm

Providing 8mm diameter

- The pitch of transverse reinforcement shall not more than the least of the following distances:

a) The least lateral dimension of the column, b = 450mm

b) 16 times the smallest diameter of longitudinal bar = $16 \times 25 = 400mm$

c) 300mm

Therefore spacing = 300mm

$\therefore$ Provide 8mm dia bars at 300mm c/c

6. **Design the column from the following details using SP 16 charts.**

Size of column = 300 × 450mm

P_u = 1200KN

M_u = 150kN-m

Use M25 and Fe415 steel. Assume *d* ' = 50mm

Provide reinforcement distributed equally on two sides.

Solution:

Given:

$$D = 450mm, b = 300mm, P_u = 1200kN = 1200 \times 10^3 N, M_u = 150kN - m = 150 \times 10^6 N - mm$$

$$f_{ck} = 25N/mm^2, f_y = 415N/mm^2, d' = 50\text{mm}$$

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{1200 \times 10^3}{25 \times 300 \times 450} = 0.36$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{150 \times 10^6}{25 \times 300 \times 450^2} = 0.099$$

$$\frac{d'}{D} = \frac{50}{450} = 0.11$$

Using SP:16 chart 32 ($f_y = 415N/mm^2, \frac{d'}{D} = 0.10$)

For $\frac{P_u}{f_{ck}bD} = 0.36$ and $\frac{M_u}{f_{ck}bD^2} = 0.099$ values , obtain value of $\frac{p}{f_{ck}} = 0.059$

Step : 2 Design of Longitudinal reinforcement

Percentage of steel, $p = 0.059 \times f_{ck} = 0.059 \times 25 = 1.475\%$

$$p = \frac{100 A_{sc}}{bD}$$

$$\therefore A_{sc} = \frac{pbD}{100} = \frac{1.475 \times 300 \times 450}{100} = 1991.25 mm^2$$

Providing 4 bars of 20mm ϕ + 2 bars of 25mm ϕ equally distributed on two sides.

Step 3 : Design of lateral ties

➢ Diameter shall not be less than

a) 1/4th diameter of the largest longitudinal bar = $\frac{1}{4} \times 25 = 6.25 mm$ say 8mm

b) 6mm

Providing 8mm diameter

➢ The pitch of transverse reinforcement shall not more than the least of the following distances:

a) The least lateral dimension of the column, b = 300mm

b) 16 times the smallest diameter of longitudinal bar = $16 \times 20 = 300 mm$

c) 300mm

Therefore spacing = 300mm

∴ Provide 8mm dia bars at 300mm c/c

7. **Determine the reinforcement of grade Fe415 steel required for a square column of size 400x400mm subjected to a factored direct load and a factored moment of 2000kN and 250kN-m respectively when the reinforcement is to be placed on two sides with nominal cover of 30mm. The concrete mix to be used is of grade M20.**

Solution:

Given:

$D = 400mm, b = 400mm, P_u = 2000kN = 2000\times10^3 N, M_u = 250kN-m = 250\times10^6 N-mm$

$f_{ck} = 20N/mm^2, f_y = 415N/mm^2$, Nominal cover = 30mm

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{2000\times10^3}{20\times400\times400} = 0.625$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{250\times10^6}{20\times400\times400^2} = 0.195$$

Assuming 25mm ϕ longitudinal bars with 30mm nominal cover,

Effective cover, $d' = 30 + \frac{25}{2} = 42.5mm$

Now, $\frac{d'}{D} = \frac{42.5}{400} = 0.106$

Using SP:16 charts 32 ($f_y = 415N/mm^2, \frac{d'}{D} = 0.10$)

For $\frac{P_u}{f_{ck}bD}=0.625$ and $\frac{M_u}{f_{ck}bD^2}=0.195$ values obtain $\frac{p}{f_{ck}}=0.20$ from chart.

Step : 2 Design of longitudinal reinforcement

Percentage of steel, $p=0.20\times f_{ck}=0.20\times 20=4\%$

$$p=\frac{100A_{sc}}{bD}$$

$$\therefore A_{sc}=\frac{pbD}{100}=\frac{4\times 400\times 400}{100}=6400mm^2$$

Providing 32mm ϕ bars

Area of one bar, $a_{sc}=\frac{\pi\times 32^2}{4}=804.24\ \text{mm}^2$

Number of bars $=\frac{A_{sc}}{a_{sc}}=\frac{6400}{804.24}=7.92$ say 8 Nos.

$\therefore$ Provide 8 bars of 32mm equally distributed on two sides.

Step : 3 Design of lateral ties

Diameter and Pitch of lateral ties:

- Diameter shall not be less than

  a) 1/4th diameter of the largest longitudinal bar = $\frac{1}{4} \times 32 = 8mm$

  b) 6mm

  Providing 8mm diameter

- The pitch of transverse reinforcement shall not more than the least of the following distances:

  a) The least lateral dimension of the column, b = 400mm

  b) 16 times the smallest diameter of longitudinal bar = $16 \times 32 = 512mm$

  c) 300mm

  Therefore spacing = 300mm

$\therefore$ Provide 8mm dia bars at 300mm c/c

8. Design a short reinforced concrete column of rectangular section to carry on ultimate load of 600kN and ultimate moment of 100kN-m, acting about an axis bisecting the depth of the column. Assume the effective length of column is 4.5 m. Width of the supported beam is 300 mm.Use M20 concrete and Fe415 steel, provide equal steel on both tension and compression sides.

Solution:

Given:

$$P_u = 600kN = 600\times10^3 N, M_u = 100kN - m = 100\times10^6 N - mm, l_{eff} = 4.5m = 4500mm,$$

$$f_{ck} = 20N/mm^2, f_y = 415N/mm^2$$

Step : 1 Check for slenderness ratio

Now, for column to be short, $\frac{l_{eff}}{D} \leq 12$

$$D \geq \frac{l_{eff}}{12} \geq \frac{4500}{12} \geq 375mm$$

Since the width of support of beam = 300 mm, therefore, provide width of column = 300 mm. Assume size of column = 300 mm × 400 mm (ie., b = 300 mm, D = 400 mm)

Step 2 : Non dimensional parameters

$$\frac{P_u}{f_{ck}bD}=\frac{600\times10^3}{20\times300\times400}=0.25$$

$$\frac{M_u}{f_{ck}bD^2}=\frac{100\times10^6}{20\times300\times400^2}=0.104$$

Assuming effective cover = $d'=40mm$

Now, $\frac{d'}{D}=\frac{40}{400}=0.10$

Using SP:16 chart 32 $\left(\frac{d'}{D}=0.10, f_y=415N/mm^2\right)$

For $\frac{P_u}{f_{ck}bD}=0.25$ and $\frac{M_u}{f_{ck}bD^2}=0.104$, value of $\frac{p}{f_{ck}}=0.04$ from chart

Step 3 : Design of Longitudinal reinforcement

Percentage of steel, $p=0.04\times f_{ck}=0.04\times20=0.8\%$

$$p=\frac{100A_{sc}}{bD}$$

$$\therefore A_{sc}=\frac{pbD}{100}=\frac{0.8\times300\times400}{100}=960mm^2$$

Area of one 20mm dia bar, $a_{sc} = \frac{\pi \times 20^2}{4} = 314.15mm^2$

Number of bars = $\frac{A_{sc}}{a_{sc}} = \frac{960}{314.15} = 3.05$ say 4

Provide 4 bars of 20mm dia as longitudinal reinforcement.

Step 4 : Design of lateral ties

Diameter and Pitch of lateral ties:

- Diameter shall not be less than
  a) 1/4th diameter of the largest longitudinal bar

  $= \frac{1}{4} \times 20 = 5mm$

  b) 6mm

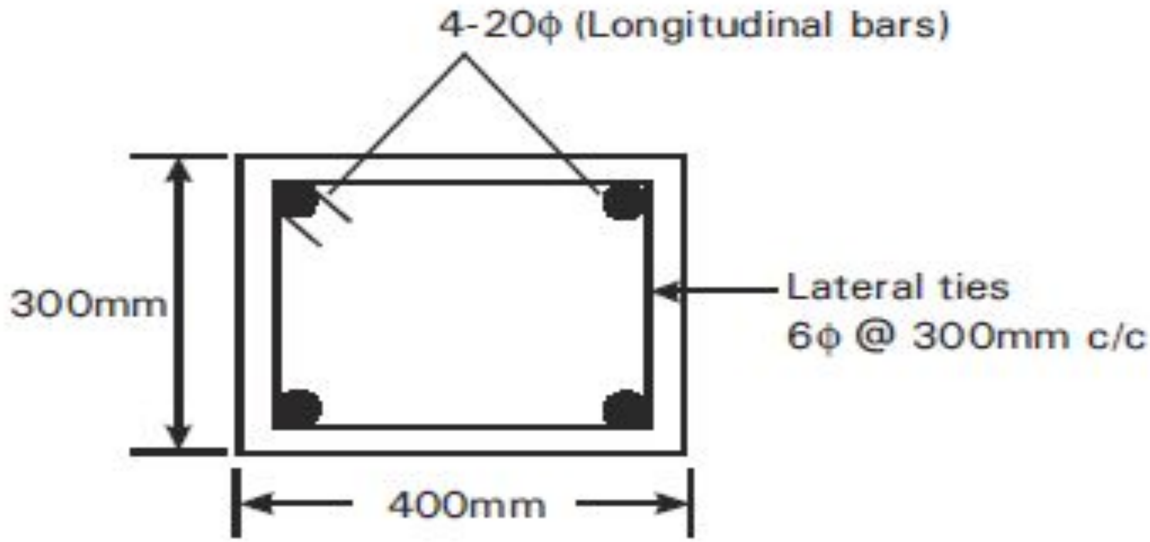
  Providing 6mm diameter

- The pitch of transverse reinforcement shall not more than the least of the following distances:
  a) The least lateral dimension of the column, b = 300mm
  b) 16 times the smallest diameter of longitudinal bar = $16 \times 20 = 320mm$
  c) 300mm

  Therefore spacing = 300mm

$\therefore$ Provide 6mm dia bars at 300mm c/c

Step 5 : Reinforcement details (C/S)



9. **Determine the reinforcement to be provided in a square column subjected to uniaxial bending with the following data :**

Size of column 500 × 500mm

Factored load = 2500 kN

Eccentricity, e = 80mm

f_{ck} **= 25 N/mm² and** f_y **= 45 N/mm²**

Arrangement of reinforcement :

(1) on two sides

(2) on four sides

Sketch the reinforcement details

Solution:

Given:

$b = D = 500$ mm, $P_u = 2500$ kN, $e = 80$mm, $f_{ck} = 25$ N/mm² and $f_y = 415$ N/mm²

Step : 1 Non dimensional parameters

$$\frac{P_u}{f_{ck}bD} = \frac{2500 \times 10^3}{25 \times 500 \times 500} = 0.4$$

$$e = \frac{M_u}{P_u} \Rightarrow M_u = P_u \times e = 2500 \times 10^3 \times 80 = 200 \times 10^6 \text{ N-mm}$$

$$\therefore M_u = 200 \times 10^6 \text{ N-mm}$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{200 \times 10^6}{25 \times 500 \times 500^2} = 0.064$$

Step : 2 Design of Longitudinal reinforcement

(1) On two sides reinforcement

Assuming effective cover, d' = 50mm

Now, $\frac{d'}{D} = \frac{50}{500} = 0.10$

Using SP:16, chart 32 ($f_y = 415\ N/mm^2$ and $\frac{d'}{D} = 0.10$)

For $\frac{P_u}{f_{ck}bD} = 0.4$ and $\frac{M_u}{f_{ck}bD^2} = 0.064$, values obtain $\frac{p}{f_{ck}} = 0.04$ from chart

$\therefore$ Percentage of steel, $p = 0.04 \times f_{ck} = 0.04 \times 25 = 1\%$

$$p = \frac{100 A_{sc}}{bD}$$

$\therefore A_{sc} = \frac{pbD}{100} = \frac{1 \times 500 \times 500}{100} = 2500\ mm^2$ (A_{sc} required)

Providing 25 mm dia bars

$$\text{No. of bars} = \frac{A_{sc\ required}}{a_{sc}} = \frac{2500}{\frac{\pi}{4} \times 25^2} = 5.09 \text{ say } 6$$

$$A_{sc_{provided}} = \frac{6 \times \pi \times 25^2}{4} = 2945.24 \text{ mm}^2 > A_{sc_{required}}$$

Hence (ok)

∴ Provide 6 bars of 25mm ϕ equally distributed on two sides

(2) On four sides reinforcement

Now using SP : 16, chart 44 $\left(f_y = 415 \text{N / mm}^2, \frac{d^1}{D} = 0.10 \right)$

For $\frac{P_u}{f_{ck} bD} = 0.4$ and $\frac{M_u}{f_{ck} bD^2} = 0.064$, values obtain $\frac{p}{f_{ck}} = 0.04$ from chart

∴ Percentage of steel, $p = 0.04 \times f_{ck} = 0.04 \times 25 = 1\%$

$$p = \frac{100 A_{sc}}{bD}$$

$$\therefore A_{sc} = \frac{pbD}{100} = \frac{1 \times 500 \times 500}{100} = 2500 \text{ mm}^2 \left(A_{sc_{required}} \right)$$

Providing 20 mm dia bars

$$\text{No. of bars} = \frac{A_{sc\,required}}{a_{sc}} = \frac{2500}{\frac{\pi}{4} \times 20^2} = 7.95 \text{ say } 8$$

$$A_{sc_{provided}} = \frac{8 \times \pi \times 20^2}{4} = 2513.27 \text{ mm}^2 > A_{sc_{required}}$$

Hence (ok)

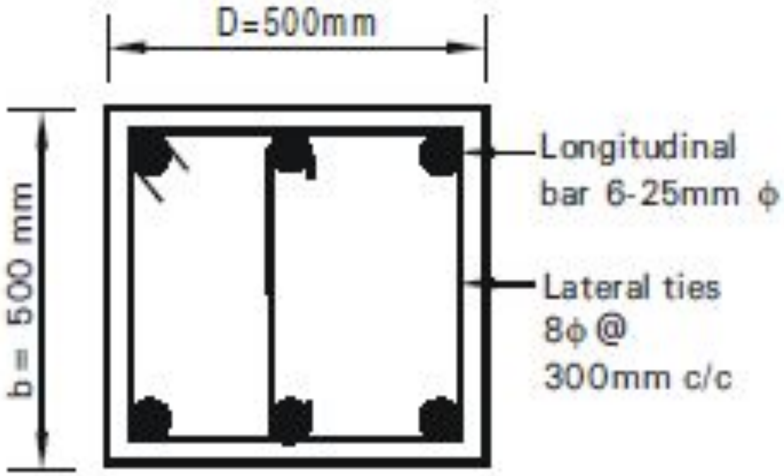
∴ Provide 8 bars of 20mm ϕ equally distributed on four sides

Step 3 : Design of lateral ties

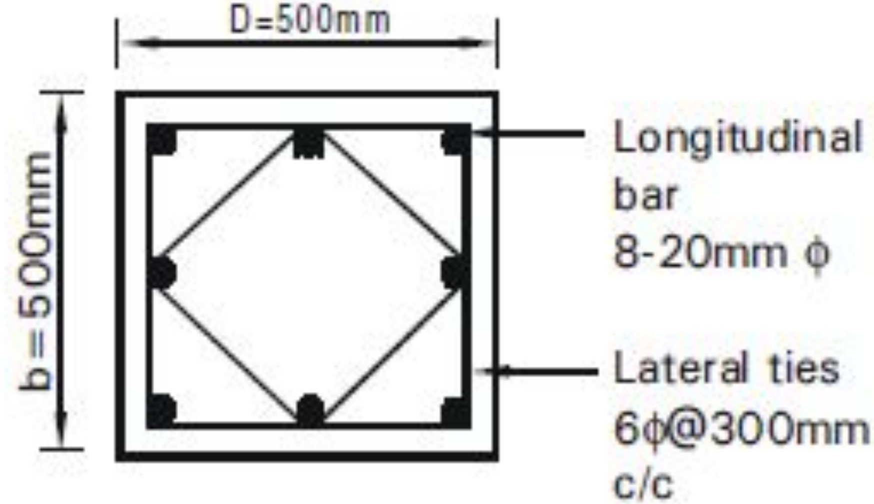
Sl. No.	Diameter and Pitch of lateral ties	Design of lateral ties	
		on two sides reinforcement	on four sides reinforcement
1	⅄ Diameter : Shall not be less then (i) $\frac{1}{4}$ × diameter of the largest longitudinal bar (ii) 6mm	(i) $\frac{1}{4} \times 25 = 6.25$mm (ii) 6mm say 8mm	(i) $\frac{1}{4} \times 20 = 5$mm (ii) 6mm say 6mm
2	⅄ Pitch of transverse reinforcement shall not more than the least of the following distances (i) The least lateral dimension of the column (ii) 16 times the smallest diameter of longitudinal bar (iii) 300mm	(i) b = 500mm (ii) $16 \times 25 = 400$mm (iii) 300mm ∴ Provide 8mm dia bars at 300mm c/c	(i) b = 500mm (ii) $16 \times 20 = 320$mm (iii) 300mm ∴ Provide 6mm dia bars at 300mm c/c

Step 4 : Reinforcement details (C/S)

(1) On two sides



(2) On four sides



10. **Design the reinforcement to be provided in a circular column with the following data :**

Diameter of column = 400 mm

Factored axial load = 1500 kN

Factored moment = 100 kN.m

Unsupported length = 2.5 m

Grade of concrete and steel = 20 N/mm^2 and 415 N/mm^2

Design the lateral reinforcement

(a) Hoop reinforcement

(b) Helical reinforcement

Solution :

Given : D = 400mm, P_u = 1500kN, M_u = 100 kN.m, l = 2.5m f_{ck} = 20N/mm^2 and f_y = 415 N/mm^2

(a) Column with Hoop reinforcement

Step 1 : Check for slenderness ratio

Assuming both ends hinged

$$\therefore l = l_{eff} = 2.5\text{m or } 2500\text{mm}$$

$$\therefore \frac{l_{eff}}{D} = \frac{2500}{400} = 6.25 < 12$$

Hence it is short column

Step 2 : Check for Eccentricity

$$\text{Eccentricity, } e = \frac{M_u}{P_u} = \frac{100 \times 10^6}{1500 \times 10^3} = 66.6\text{mm}$$

$$\text{Minimum eccentricity, } e_{\min} = \frac{l}{500} + \frac{D}{30} = \frac{2500}{500} + \frac{400}{30} = 18.33\text{mm} > 20\text{mm}$$

$$\therefore e > e_{\min}$$

Hence, it is designed as short column subjected to axial load and bending

Step 3 : Non dimensional parameters

$$\frac{P_u}{f_{ck} D^2} = \frac{1500 \times 10^3}{20 \times (400)^2} = 0.468$$

$$\frac{M_u}{f_{ck} D^3} = \frac{100 \times 10^6}{20 \times (400)^3} = 0.078$$

Assuming effective cover $d^1 = 50\text{mm}$

$$\text{Now, } \frac{d^1}{D} = \frac{50}{400} = 0.125 \approx 0.15$$

Using SP : 16, chart 57 $\left(f_y = 415\text{N / mm}^2, \frac{d^1}{D} = 0.15 \right)$

For $\frac{P_u}{f_{ck} D^2} = 0.468$, and $\frac{M_u}{f_{ck} D^3} = 0.078$, values obtain $\frac{p}{f_{ck}} = 0.16$ from chart

Step 4 : Design of Longitudinal reinforcement

Percentage of steel, $p = 0.16 \times f_{ck} = 0.16 \times 20 = 3.2\% < 4\%$ (maximum steel, ok)

$$A_{sc} = \frac{p\pi D^2}{400} = \frac{3.2 \times \pi \times (400)^2}{400} = 4021.23\text{mm}^2 \left(A_{sc_{required}}\right)$$

Providing 32mm dia bars

$$\text{No. of bars} = \frac{A_{sc_{required}}}{a_{sc}} = \frac{4021.23}{\frac{\pi}{4} \times 32^2} = 5 \text{ say } 6$$

$$A_{sc_{provided}} = \frac{6 \times \pi \times 32^2}{4} = 4825.48\text{mm}^2 > A_{sc_{required}}$$

Hence (ok)

$\therefore$ Provide 6 bars of 32mm ϕ

Step 5 : Design of lateral ties (Hoop reinforcement)

$\ngtr$ Diameter shall not be less then

(a) $\frac{1}{4} \times 32 = 8mm$

(b) 6mm say 8mm

$\ngtr$ Pitch of transverse reinforcement shall not more than the least of the following distances:

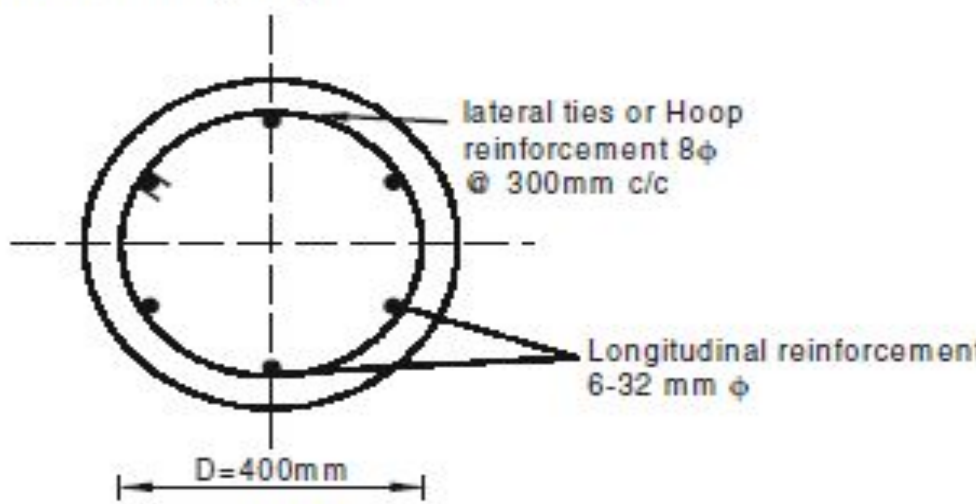
(i) The least lateral dimension of the column, $D = 400mm$

(ii) 16 times the smallest diameter of longitudinal bar = $16 \times 32 = 512$ mm

(iii) 300mm

$\therefore$ Provided 8mm ϕ bars at 300mm c/c

Step 6 : Reinforcement details (C/S)



WORKED EXAMPLES ON DESIGN OF COLUMN SUBJECTED TO COMBINED AXIAL COMPRESSION LOAD AND BIAXIAL BENDING

✸ Using SP-16 Method

1. **Design the reinforcements in a short column subjected to biaxial bending, with the following data :**

- **Size of column - 400 × 600 mm**
- **Concrete Mix - M20**
- **Characteristic strength of reinforcement - 415 N/mm²**
- **Factored load, P_u - 1600 kN**
- **Factored moment acting parallel to the larger dimension, M_{ux} - 120 kN.m**
- **Factored moment acting parallel to the shorter dimension, M_{uy} - 90 kN.m**

Solution :

Given : $b = 400$mm, $D = 600$mm, $P_u = 1600$ kN, $M_{ux} = 120$ kN.m, $M_{uy} = 90$ kN.m, $f_{ck} = 20$ N/mm² and $f_y = 415$ N/mm²

Step 1 : Check for Eccentricity

$$e_x = \frac{M_{ux}}{P_u} = \frac{120\times10^6}{1600\times10^3} = 75 \text{ mm}$$

$$e_y = \frac{M_{uy}}{P_u} = \frac{90\times10^6}{1600\times10^3} = 56.25 \text{ mm}$$

Minimum Eccentricity, $e_{min} = 20$mm

$$\therefore e_x \text{ and } e_y > e_{min}$$

Hence column is designed as short column subjected to axial load and biaxial bending.

Step 2 : Find the moment capacities M_{ux_1} and M_{uy_1}

Trial - 1 : As a first trial, assume the reinforcement percentage, $p = 1.2\%$ (Assume reinforcement is distributed equally on four sides)

$$\frac{p}{f_{ck}} = \frac{1.2}{20} = 0.06$$

- **Uniaxial moment capacity of the section about x-axis (major axis) :**

  Assuming 22 mm bars with 40 mm cover,

  $d' = 40 + \left(22/2\right) = 51$ mm, say $d' = 50$ mm

  $$\frac{d'}{D} = \frac{50}{600} = 0.0833$$

  Chart for $\frac{d'}{D} = 0.10$ will be used.

  $$\frac{P_u}{f_{ck}bD} = \frac{1600 \times 10^3}{20 \times 400 \times 600} = 0.333$$

  Referring to chart-44 of SP-16

  $$\frac{M_u}{f_{ck}bD^2} = 0.1$$

  $\therefore M_u = M_{ux_1} = 0.1 \times 20 \times 400 \times (600)^2 = 288 \times 10^6$ N.mm or 288 kN.m

➢ **Uniaxial moment capacity of the section about *y*-axis (minor axis) :**

$$\frac{d'}{D} = \frac{50}{400} = 0.125$$

Chart for $\frac{d'}{D} = 0.15$ will be used.

Referring to chart-45 of SP-16

$$\frac{M_u}{f_{ck} b^2 D} = 0.09$$

$$\therefore M_u = M_{uy_1} = 0.09 \times 20 \times (400)^2 \times 600 = 172.8 \text{ kN.m}$$

Step 3 : Check for safety under biaxial bending

Referring to chart - 63 of SP-16, corresponding to $p = 1.2\%$, $f_y = 415$ N/mm² and $f_{ck} = 20$ N/mm².

$$\frac{P_{uz}}{A_g} = 12.5 \text{ N/mm}^2 \qquad | A_g = b \times D$$

$$\therefore P_{uz} = 12.5\, A_g = 12.5 \times 400 \times 600 = 3000\ kN$$

$$\frac{P_u}{P_{uz}} = \frac{1600}{3000} = 0.533$$

$$\frac{M_{ux}}{M_{ux_1}} = \frac{120}{288} = 0.416$$

$$\frac{M_{uy}}{M_{uy_1}} = \frac{90}{172.8} = 0.520$$

Referring to chart - 64 of SP - 16, the permissible value of $\frac{M_{ux}}{M_{ux_1}}$ corresponding to the above values of $\frac{M_{uy}}{M_{uy_1}}$ and $\frac{P_u}{P_{uz}}$ is equal to 0.75.

The actual value of 0.416 is lesser than the value read from the chart. i.e., $0.75 > 0.416$

Hence the column is safe under biaxial bending

Note : Trial - 2 is not required for this problem ($\because 0.75 > 0.416$). Hence the user can use $p = 1\%$ steel to check safety against biaxial bending.

Step 4 : Design of Longitudinal reinforcement

$$A_{sc} = \frac{pbD}{100} = \frac{1.2 \times 400 \times 600}{100} = 2880\text{mm}^2 \left(A_{sc_{required}} \right)$$

Assuming 22mm dia bars

$$\text{Area of one bar, } a_{sc} = \frac{\pi}{4} \times 25^2 = 380.13\text{mm}^2$$

$$\therefore \text{No. of bars} = \frac{A_{sc}}{a_{sc}} = \frac{2880}{380.13} = 7.57 \text{ say } 8$$

$$A_{sc_{provided}} = 8 \times \frac{\pi}{4} \times 22^2 = 3041.06 \text{ mm}^2 > A_{sc_{required}}, \text{ Hence (ok)}$$

$\therefore$ Provide 8 bars of 22mm dia bars and distributed equally on all faces (i.e., four sides)

Step 5 : Design of lateral ties

≯ Diameter shall not be less than

(i) $\frac{1}{4} \times 22 = 5.5$ mm

(ii) 6mm, say 8 mm

≯ Pitch of transverse reinforcement shall not more than

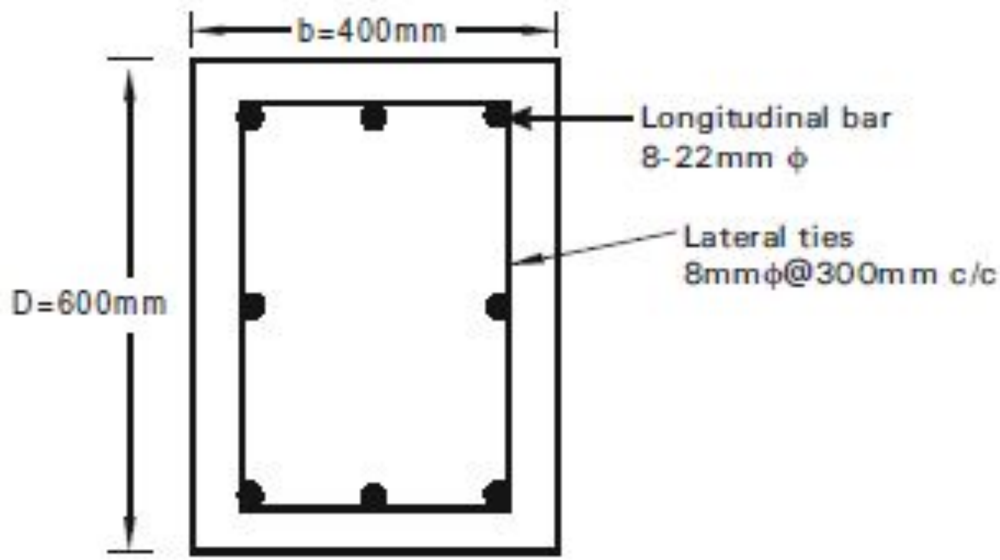
(i) least lateral dimension, $b = 400$mm

(ii) $16 \times \phi = 16 \times 22 = 352$mm

(iii) 300mm

$\therefore$ Provide 8mm dia bars at 300mm c/c

Step 6 : Reinforcement details



UNIT:2-Limit State Design of Footings

* Introduction

Reinforced columns are supported by the footings which are located below the ground level and is referred as the foundation structure. The structural design of the footing, which includes the design of the depth and reinforcements, is done for factored loads using the relevant safety factors applicable for the limit state of collapse. Footings are designed for flexure and shear (both one-way and two way action). The design is more or less similar to that of beams and two way slabs supported on columns. Additional design considerations being the transfer of force from the column to the footing and also safety against sliding and overturning when horizontal forces acting on the structure.

Due to the loads and soil pressure, footings develop bending moments and shear forces. Calculations are made as per the guidelines suggests in IS : 456 - 2000 to resist the internal forces.

TYPES OF FOUNDATION

Foundations are classified into two types

1. Shallow Foundations
   - Isolated footing
   - Combined footing
   - Strap footing
   - Mat / Raft foundation
   - Wall footing
2. Deep foundations
   - Pile foundations
   - Pier foundations
   - Well foundations
   - Caisson foundations

Isolated footings

Footings which are provided under each column independently are called as isolated footings.

They may be square, rectangular or circular in plan. Isolated footings comprise of a thick slab which may be flat or sloped or stepped.

Isolated footings are ideally provided when loads are small and the soil is not very poor i.e., its bearing capacity (SBC) is sufficient. It is preferable to provide rectangular or square foundations. Isolated square or rectangular footings are of two types :

(a) Uniform thickness footings

(b) Tapered thickness footings

(a) (b)

The isolated footings essentially consists of bottom slab. The bottom of the slab is reinforced with steel mesh to resist the two internal forces namely bending moment and shear force.

DESIGN STEPS FOR ISOLATED FOOTING (SQUARE AND RECTANGULAR COLUMN FOOTING WITH AXIAL LOAD)

Given : Column load (P), size of column ($b \times D$), Bearing capacity of soil (SBC), Grade of concrete and steel.

Required : size of footing ($L \times B$), Design for shear (check for one way and two way shear), Design for flexure and check for development length

Design steps :

1. **Load calculations :**

$$\text{Column axial load } P = \_\_\_\_ \text{ kN}$$

Self weight of footing

$$\frac{10}{100} \times P = 0.1P$$

(Assume 10% of column load $= 0.1P$ kN

$$\text{Total load, } W = P + 0.1P$$

Note : 10 to 15% of load from columns may be taken as self weight of footing for determining the area of footing required

2. **Area of footing required**

$$A_{\text{required}} = \frac{\text{Total load}}{\text{SBC of soil}} = \frac{W}{\text{SBC}}$$

3. **Size of footing and Area provided**

1) If square footing, then size of footing, $L = B = \sqrt{A_{\text{required}}}$ (round off to nearest value)

2) If Rectangular footing

A = Area of footing

$A = L \times B$

assume one side either L or B

$$\therefore L = \frac{A_{\text{required}}}{B} \text{ or } B = \frac{A_{\text{required}}}{L} \text{ (round off to nearest value)}$$

$$\text{Now, } A_{\text{provided}} = L \times B$$

4. Net upward pressure (q)

$$q = \frac{\text{Column axial load}}{\text{Area provided}} = \frac{P}{A_{\text{provided}}}$$

if $q <$ SBC of soil (Given), Hence safe (ok)

if $q >$ SBC of soil (Given), not safe, Hence Increase the area of footing

5. Bending moment calculation (M_u)

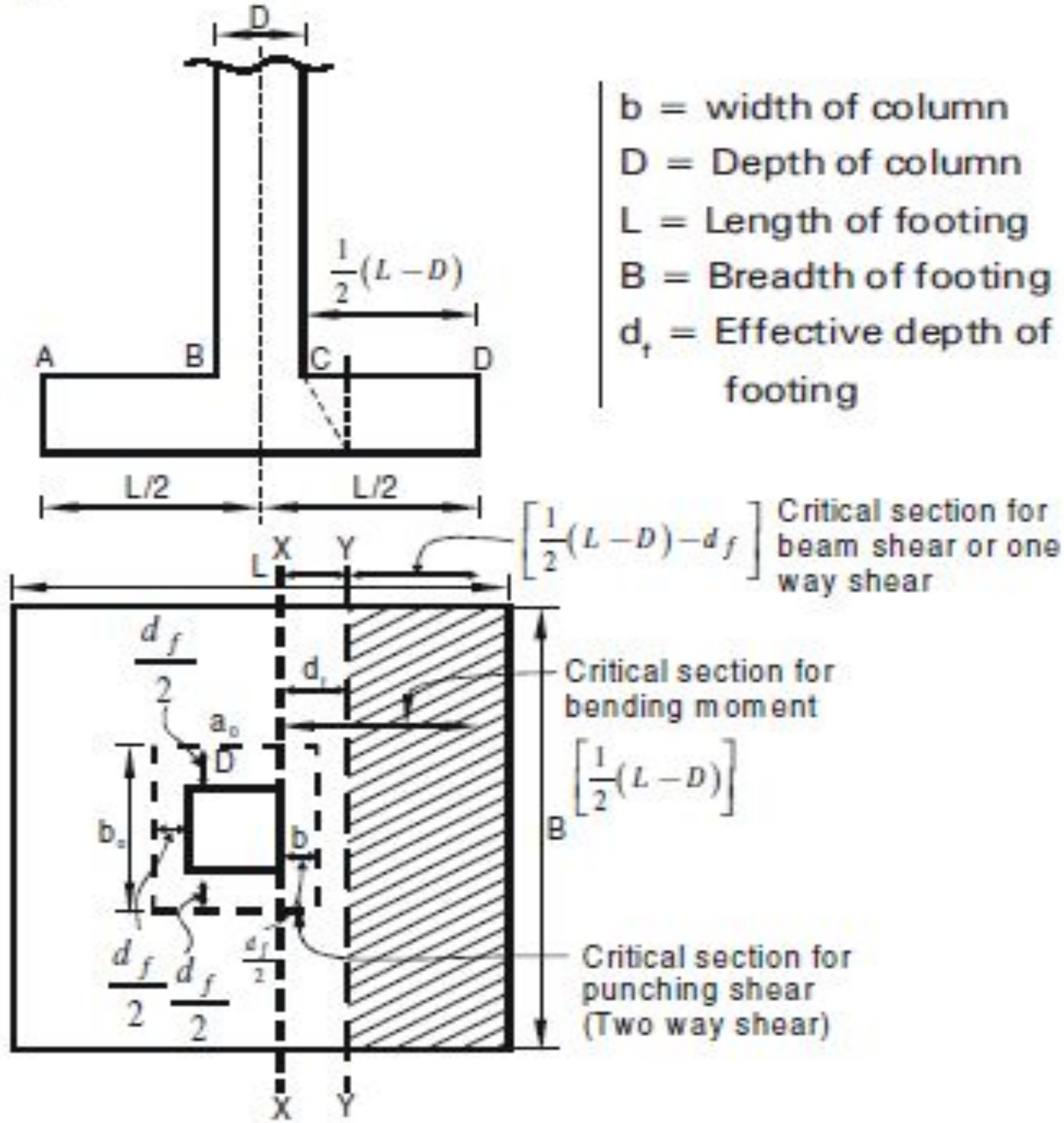
1) Square footing

$$a_o = D + \frac{d_f}{2} + \frac{d_f}{2}$$

$$a_o = D + d_f$$

$$b_o = b + \frac{d_f}{2} + \frac{d_f}{2}$$

$$b_o = b + b_f$$



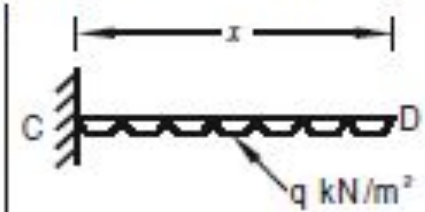
b = width of column
D = Depth of column
L = Length of footing
B = Breadth of footing
d_f = Effective depth of footing

Fig. 5.13 : Square footing (Plan and L/S)

Maximum bending moment will take place about section x–x near the face of the column.

The unit pressure q causing bending of cantilever of length 'CD'

i.e., $\frac{1}{2}(L–D)$ ($\because$ critical section for bending moment)



$$\therefore M = q \times B \times \left[\frac{1}{2}(L-D)\right] \times \frac{1}{2} \times \left[\frac{1}{2}(L-D)\right]$$

$$= q \times B \times \left[\frac{L-D}{2}\right]\left[\frac{L-D}{4}\right]$$

$$\boldsymbol{M = q \times B \times \frac{1}{2}\left(\frac{L-D}{2}\right)^2 \text{ or } M = \frac{qB}{8}(L-D)^2}$$

Factored moment, $M_u = 1.5 \times M$

$$M = q \times x \times x/2$$

$$M = q \times \frac{x^2}{2}$$

Consider width of footing

$$\Rightarrow M = q \times B \times \frac{x^2}{2}$$

Here $x = \frac{1}{2}(L–D)$

$$a_0 = D + d_f$$
$$b_0 = b + b_f$$

2) Rectangular footing

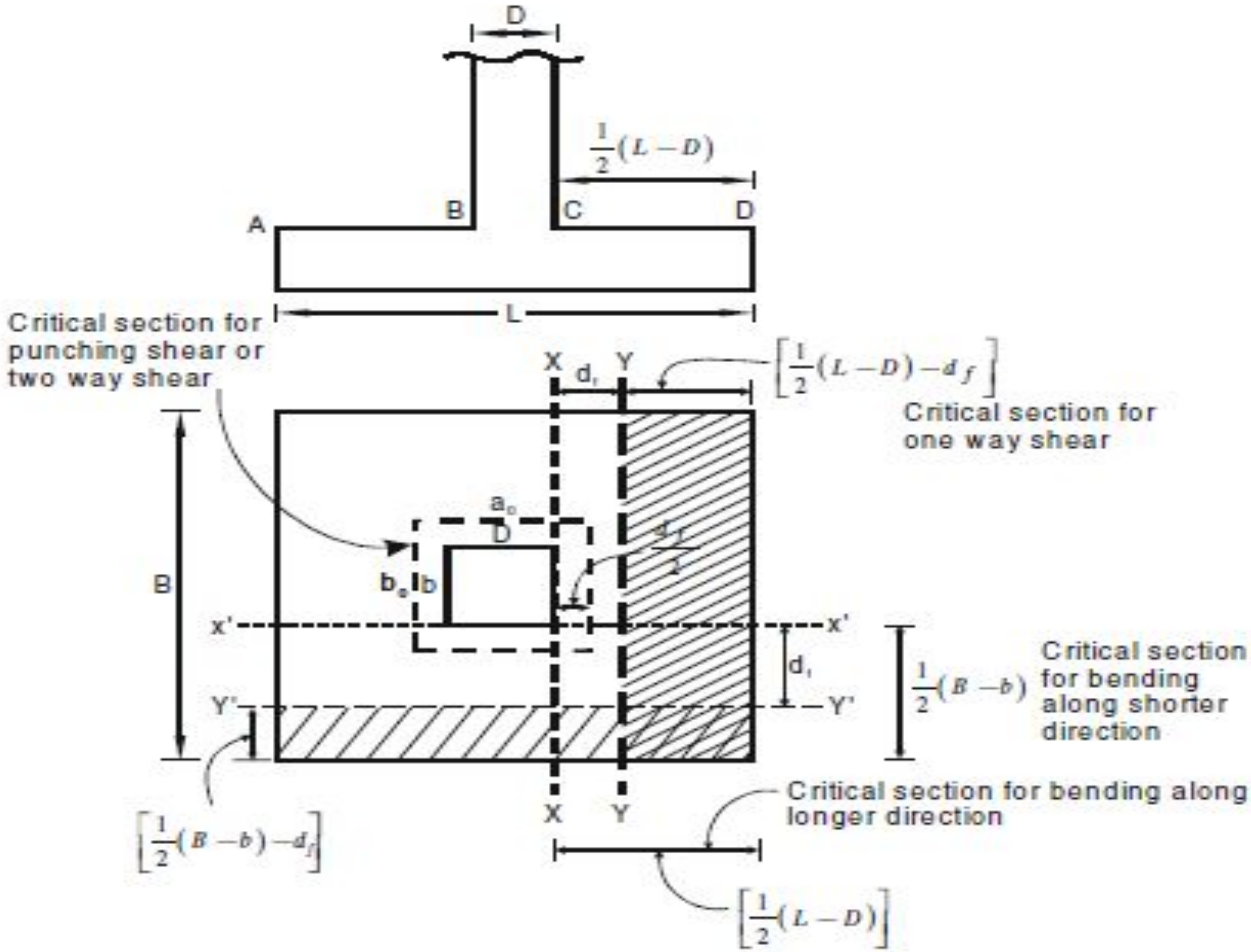


Fig. 5.14 : Rectangular footing (Plan and L/S)

Bending moment M_1 about section x–x is given by

➢ **Along longer direction**

$$M_1 = q \times B \times \left[\frac{1}{2}(L-D)\right] \times \frac{1}{2} \times \left[\frac{1}{2}(L-D)\right]$$

$$\boldsymbol{M_1 = q \times B \times \frac{1}{2}\left(\frac{L-D}{2}\right)^2 \text{ or } M_1 = \frac{qB}{8}(L-D)^2}$$

Factored moment, $M_{u_1} = 1.5 \times M_1$

Now, Bending moment M_2 about section x^1–x^1 is given by

➢ **Along shorter direction**

$$M_2 = q \times L \times \left[\frac{1}{2}(B-b)\right] \times \frac{1}{2} \times \left[\frac{1}{2}(B-b)\right]$$

$$\boldsymbol{M_2 = q \times L \times \frac{1}{2}\left(\frac{B-b}{2}\right)^2 \text{ or } M_2 = \frac{qL}{8}(B-b)^2}$$

Forced moment, $M_{u_2} = 1.5 \times M_2$

6. **Effective depth required**

$$\text{Equating } M_u = M_{u_{lim}} \text{ (Balanced section)}$$

$$M_{u_{lim}} = 0.138\, f_{ck} B d_f^2 \text{ \_\_\_ for Fe 415}$$

$$\therefore d_{f_{required}} = \sqrt{\frac{M_u}{0.138 f_{ck} B}}$$

| d^1 = Effective cover

$$D_f = d_f + d^1$$

Note : Increase footing depth (d_f) 1.75 to 2 times due to shear consideration

7. **Design of steel reinforcement or Area of steel reinforcement**

By referring IS : 456 - 2000, Annex - G

G - 1.1 (b)

$$M_u = 0.87 f_y A_{st} d_f \left[1 - \left(\frac{f_y A_{st}}{f_{ck} B d_f}\right)\right]$$

Solving above expression using quadratic equation, calculate 'A_{st}'

$$\text{Area of steel per meter} = \frac{A_{st}}{B} \text{ in mm}^2$$

| B = width of footing in 'm'

Assume suitable diameter of bars (eg : ϕ = 8mm, 10mm, 12mm, 16mm.....)

$$\text{Area of one bar, } a_{st} = \frac{\pi \times \phi^2}{4}$$

$$\text{Spacing of reinforcement, } S = \frac{1000 a_{st}}{A_{st}}$$

Note : For rectangular footing calculate area of steel along both directions using M_{u_1} and M_{u_2}

8. Design for shear

1) Check for one way shear

The critical section is taken at a distance of d_f (Effective depth) away from the face of column as shown in fig. 5.15.

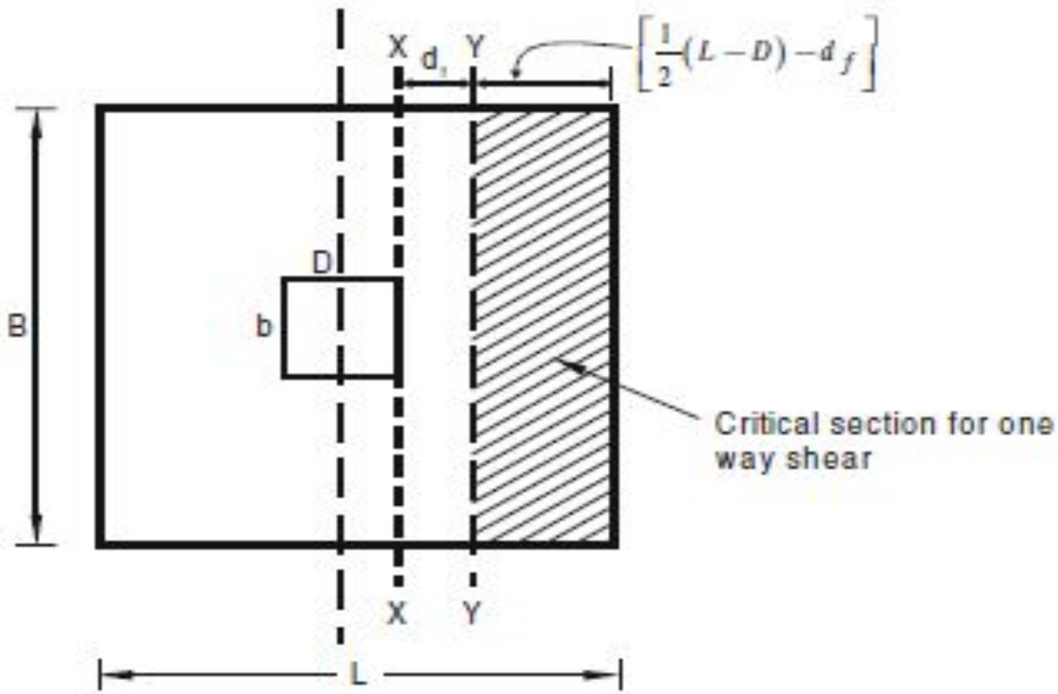


Fig. 5.15 : Plan of footing (one way shear)

∴ Shear force at the critical section is given by

$$V = q \times B \times \text{critical section for one way shear}$$

$$V = q \times B \times \left[\frac{1}{2}(L-D)-d_f\right]$$

- Factored shear force, $V_u = 1.5 \times V$
- Nominal shear stress, $\tau_v = \frac{V_u}{Bd_f}$
- Percentage of steel, $p_t = \frac{100\,A_{st}}{Bd_f}$
- Design of shear strength of concrete, τ_c

  By referring IS : 456 - 2000, Table No. 19

  Using f_{ck} and p_t, Note down the value of 'τ_c' from table No. 19
- Permissible shear stress = $k\tau_c$

  For solid slabs, the design shear strength for concrete shall be $k\tau_c$

  Where, k is depends on overall depth (D_f) of slab or footing

  Refer IS : 450 - 2000, clause 40.2.1.1

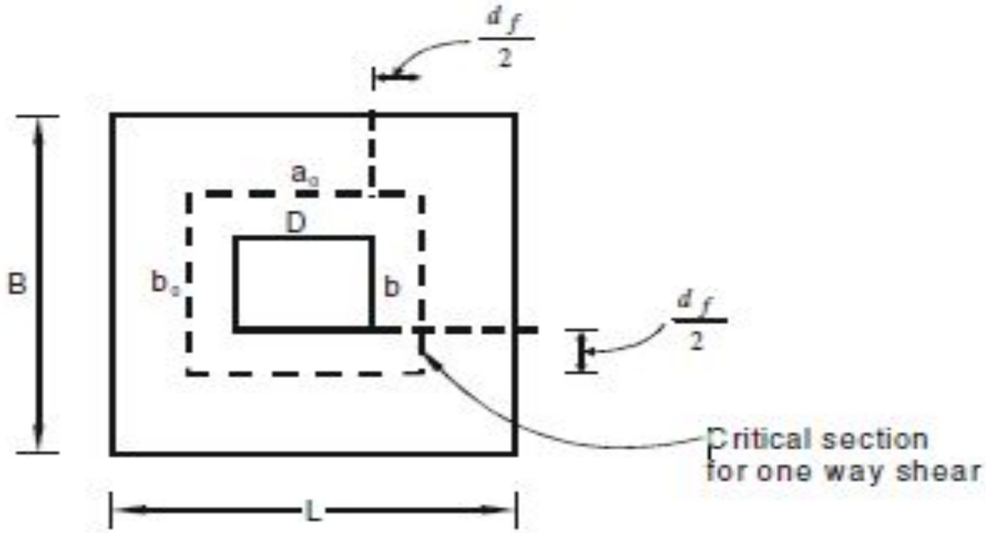
- Comparisons

  if $k\tau_c > \tau_v$ — Design is safe against one way shear

  if $k\tau_c < \tau_v$ — Design is not safe, Hence increase the depth of footing.

2) **Check for two way shear or punching shear**

The critical section is taken at a distance $\frac{d_f}{2}$ or $0.5\,d_f$ away from the face of column as shown in fig. 5.16.



$$a_0 = D + \frac{d_f}{2} + \frac{d_f}{2} = D + d_f$$

$$b_0 = b + \frac{d_f}{2} + \frac{d_f}{2} = b + d_f$$

Fig. 5.16 : Plan of footing (Two way shear)

∴ Shear force at the critical section is given by

$$V = q \times [A_{prov} - a_o \times b_o] \qquad |A_{provided}$$

- Factored shear force, $V_u = 1.5 \times V$
- Nominal shear stress, $\tau_v = \dfrac{V_u}{2(a_o + b_o) \times d_f}$
- Design of shear strength of concrete τ_c

$$\tau_c = 0.25\sqrt{f_{ck}} \quad \text{— from IS - 456 : 2000, clause 31.6.3.1}$$

- Permissible shear stress $= k_s \tau_c$

where $k_s = (0.5 + \beta_c)$, but not greater than 1

$$\beta_c = \frac{b}{D} = \frac{\text{short side of column}}{\text{long side of column}}$$

- Comparisons

  if $\tau_c > \tau_v$ — Design is safe against two way shear

  if $\tau_c < \tau_v$ — Design is not safe (Hence increase the depth of footing)

9. Check for development length of bars

Refer IS : 456 - 2000, clause 26.2.1

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}, \quad \text{where } L_d = \text{development length}$$

$$\Rightarrow L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} \qquad \sigma_s = 0.87 f_y$$

$$\text{Length available} = \left\{\frac{1}{2}[B-b] - \text{side cover}\right\} > L_d$$

Hence ok

10. Reinforcement details

(1) Plan of footing

(2) Longitudinal section (*L/S*) of footing

WORKED EXAMPLES ON DESIGN OF ISOLATED FOOTINGS - SQUARE AND RECTANGULAR COLUMN FOOTINGS WITH AXIAL LOAD

1. **Design a square footing to carry a column load of 1200kN from a 400mm square column. The bearing capacity of soil is 120kN/m² (12 tons/m²). Use M20 concrete and Fe415 steel.**

Solution :

Given : $P = 1200\text{kN}$, $b = D = 400\text{mm}$, SBC = 120 kN/m² $f_{ck} = 20\text{N/mm}^2$ and $f_y = 415\ \text{N/mm}^2$

Step 1 : Load calculations

Column axial load, $P = 1200$ kN

Self weight of footing $= 0.1 \times 1200 = 120$ kN

(Assume 10% of column load)

$\therefore$ Total load $W = 1320$ kN

Step 2 : Area of footing required

$$A_{\text{required}} = \frac{\text{Total load}}{\text{SBC of soil}} = \frac{W}{\text{SBC}} = \frac{1320}{120} = 11\text{m}^2$$

Step 3 : Size of footing and Area provided

For square footing, size of footing, $L = B = \sqrt{A_{\text{required}}} = \sqrt{11} = 3.31\text{m}$

$\therefore$ Provide 3.5 × 3.5m square footing

$\therefore L = B = 3.5\text{m}$

$\therefore$ Total area or Area provided, $A_{\text{provided}} = L \times B = 3.5 \times 3.5 = 12.25\text{m}^2$

Cont…

Step 4 : Net upward pressure (q)

$$q = \frac{\text{Column axial load}}{\text{Area provided}} = \frac{P}{A_{\text{provided}}} = \frac{1200}{12.25} = 97.96\ \text{kN/m}^2 < 120\ \text{kN/m}^2\ \text{(SBC)}$$

Hence (ok)

Step 5 : Bending moment calculation (M_u)

$$M = q \times B \times \frac{1}{2}\left(\frac{L-D}{2}\right)^2$$

$$= 97.96 \times 3.5 \times \frac{1}{2}\left(\frac{3.5-0.4}{2}\right)^2 \qquad | \, b = D = 400\text{mm or } 0.4\text{m}$$

$$M = 411.86\ \text{kN.m}$$

Factored moment, $M_u = 1.5 \times M = 1.5 \times 411.86 = 617.8$ kN.m or 617.8×10^6 N.mm

Step 6 : Effective depth required

Equating $M_u = M_{u_{\lim}}$ (Balanced section)

$$M_{u_{\lim}} = 0.138\ f_{ck} b d_f^2 \text{ — for Fe 415}$$

$$d_f = \sqrt{\frac{M_u}{0.138 f_{ck} B}} = \sqrt{\frac{617.8 \times 10^6}{0.138 \times 20 \times 3500}} = 252.89\text{mm} \qquad | B = 3.5\text{m or } 3500\text{mm}$$

Now increase footing depth (d_f) 1.75 to 2 times due to shear consideration

$$\therefore d_f = 1.75 \times 252.89 = 442.55 \text{ say } 500\text{mm}$$

Overall depth of footing, $D_f = d_f + d^1$ Assume 50mm effective cover

$$= 500 + 50$$ i.e., $d^1 = 50\text{mm}$

$$D_f = 550\text{mm}$$

Step 7 : Design of steel reinforcement or Area of steel reinforcement

By referring IS : 456 - 2000, G-1.1 (b)

$$M_u = 0.87 f_y A_{st} d_f \left[1 - \left(\frac{f_y A_{st}}{f_{ck} B d_f}\right)\right]$$

$$617.8 \times 10^6 = 0.87 \times 415 \times 500 A_{st} \left[1 - \left(\frac{415 A_{st}}{20 \times 3500 \times 500}\right)\right]$$

$$617.8 \times 10^6 = 180525 A_{st} - 2.14 A_{st}^2$$

$$\therefore 2.14 A_{st}^2 - 180525 A_{st} + 617.8 \times 10^6 = 0$$

By solving using quadratic equation, we get

$$\therefore \quad A_{st_{\text{required}}} = 3573.63\text{mm}^2$$

$$\text{Area of steel per meter} = \frac{3573.63}{B} = \frac{3573.63}{3.5} = 1021.03 \text{ mm}^2$$

Assume 12mm dia bars

$$\text{Area of one bar, } a_{st} = \frac{\pi \times 12^2}{4} = 113.09\text{mm}^2$$

$$\therefore \text{Spacing of reinforcement, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 113.09}{1021.03} = 110.75\text{mm}$$

∴ Provide 12mm dia bars at 100mm c/c along shorter and longer direction (both ways)

$$A_{st_{\text{provided}}} = \frac{1000 \times 113.09}{100} = 1130.9\text{mm}^2 \text{ per meter}$$

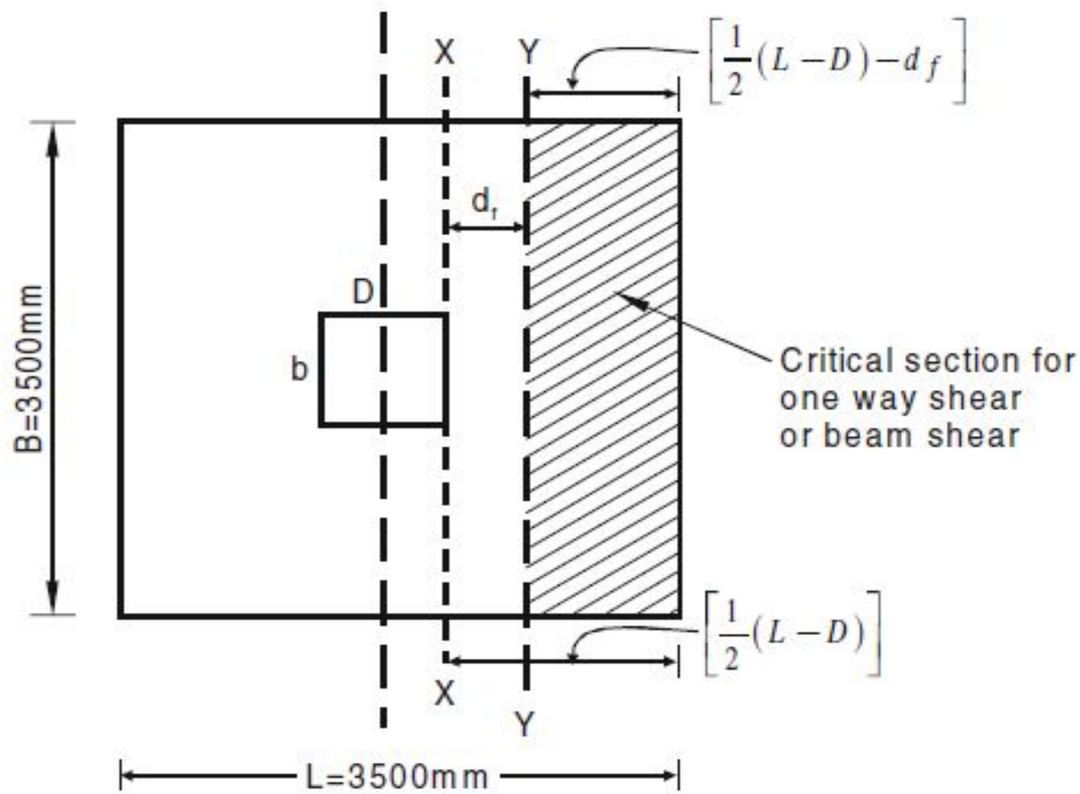
$$\text{Total } A_{st_{\text{provided}}} = 3.5 \times 1130.9 = 3958.15\text{mm}^2 > A_{st_{\text{required}}}$$

Hence (ok)

Step 8 : Design for shear

1. Check for one way shear

The critical section is taken at a distance of 'd_f' away from the face of column.



b=400mm = 0.4m
D=400mm = 0.4m

∴ Shear force at the critical section is given by

$$V = q \times B \times \text{critical section for one way shear}$$

$$= \; q \times B \times \left[\frac{1}{2}(L-D) - d_f\right]$$

$$= \; 97.96 \times 3.5 \times \left[\frac{1}{2}(3.5-0.4) - 0.5\right]$$

$$V \; = \; 360 \text{ kN}$$

- Factored shear force, $V_u = 1.5 \times V = 1.5 \times 360 = 540$ kN
- Nominal shear stress, $\tau_v = \dfrac{V_u}{Bd_f} = \dfrac{540 \times 10^3}{3500 \times 500} = 0.308 \text{ N/mm}^2$
- Percentage of steel, $p_t = \dfrac{100 A_{st}}{Bd_f} = \dfrac{100 \times 3958.15}{3500 \times 500} = 0.22\%$
- Deign of shear strength of concrete, τ_c

  By referring IS : 456 - 2000, Table No. 19, using f_{ck} - 20 N/mm^2 and $p_t = 0.22\%$

  Note down the value 'τ_c'

p_t (%)	τ_c (N/mm^2)
0.15	0.28
0.22	?
0.25	0.36

By using interpolation method, we get

$$\tau_c = 0.336 \text{ N/mm}^2$$

- Permissible shear stress = $k\tau_c$

Using IS : 456 - 2000, clause 40.2.1.1

$k = 1$ ($\because k = 1$ for overall depth of footing more than 300mm)

$$\Rightarrow k\tau_c = 1 \times 0.336 = 0.336 \text{ N/mm}^2$$

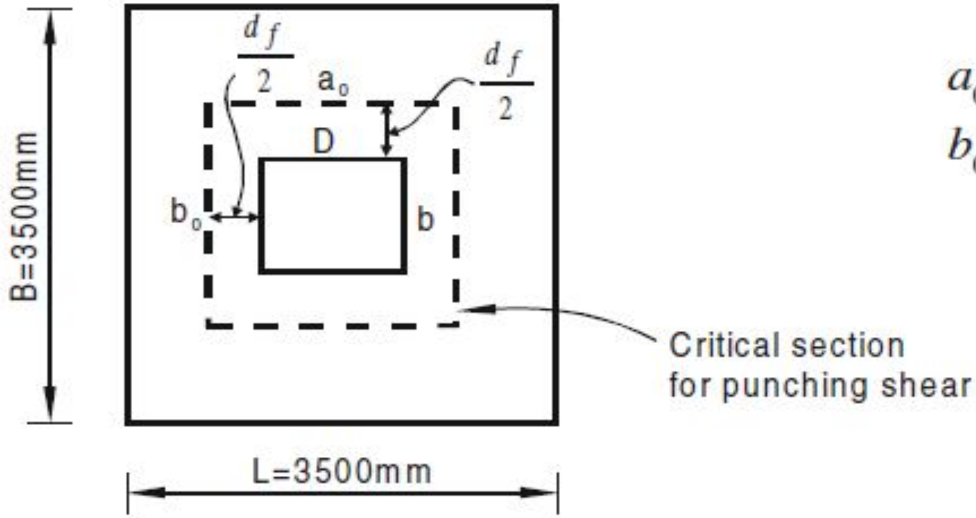
- Comparisons

$k\tau_c > \tau_v$ — Design is safe against one way shear

Hence stresses are within permissible limits

2. Check for two way shear or punching shear

The critical section is taken at a distance $\frac{d_f}{2}$ or 0.5 d_f away from the face of column.



$$a_0 = D + d_f = 0.4 + 0.5 = 0.9\text{m or }900\text{mm}$$
$$b_0 = b + d_f = 0.4 + 0.5 = 0.9\text{m or }900\text{mm}$$

∴ Shear force at the critical section is given by

$$V = q \times [A_{prov} - a_o \times b_o] \qquad |A_{provided}$$
$$= 97.96 \times [12.25 - 0.9 \times 0.9]$$
$$V = 1120.66 \text{ kN}$$

- Factored shear force, $V_u = 1.5 \times V = 1.5 \times 1120.66 = 1681$ kN
- Nominal shear stress, $\tau_v = \dfrac{V_u}{2(a_o + b_o) \times d_f}$

$$= \frac{1681 \times 10^3}{2(900+900) \times 500} = 0.93 \text{ N/mm}^2$$

- Design shear strength of concrete, τ_c

$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.11\ \text{N/mm}^2$ | from IS 456-2000, clause 31.6.3.1

- Permissible shear stress = $k_s \tau_c$

$$k_s = (0.5 + \beta_c), \text{ but not greater than } 1$$

$$\text{where } \beta_c = \frac{\text{Short side of column}}{\text{Long side of column}} = \frac{b}{D} = \frac{0.4}{0.4} = 1$$

$$\therefore k_s = (0.5 + 1) = 1.5 \not> 1$$

$$\Rightarrow k_s = 1$$

- Comparisons,

$$k_s \tau_c = 1 \times 1.11 = 1.11\ \text{N/mm}^2 > \tau_v$$

Hence Design is safe against two way shear

Step 9 : Check for development length of bars

Refer IS : 456 - 2000, clause 26.2.1

$$L_d = \frac{\phi \sigma_s}{4\tau_{bd}} \qquad \phi = 12\text{mm}$$

where $\sigma_s = 0.87 f_y$ Here $\tau_{bd} = 1.2$ for M20 concrete as per clause 26.2.1.1

For deformed bars (Fe 415 and Fe 500), values shall be increased by 60 percent. (i.e., 1.6)

$$L_d = \frac{0.87 \times 415}{4 \times 1.2 \times 1.6}\phi = 47\phi$$

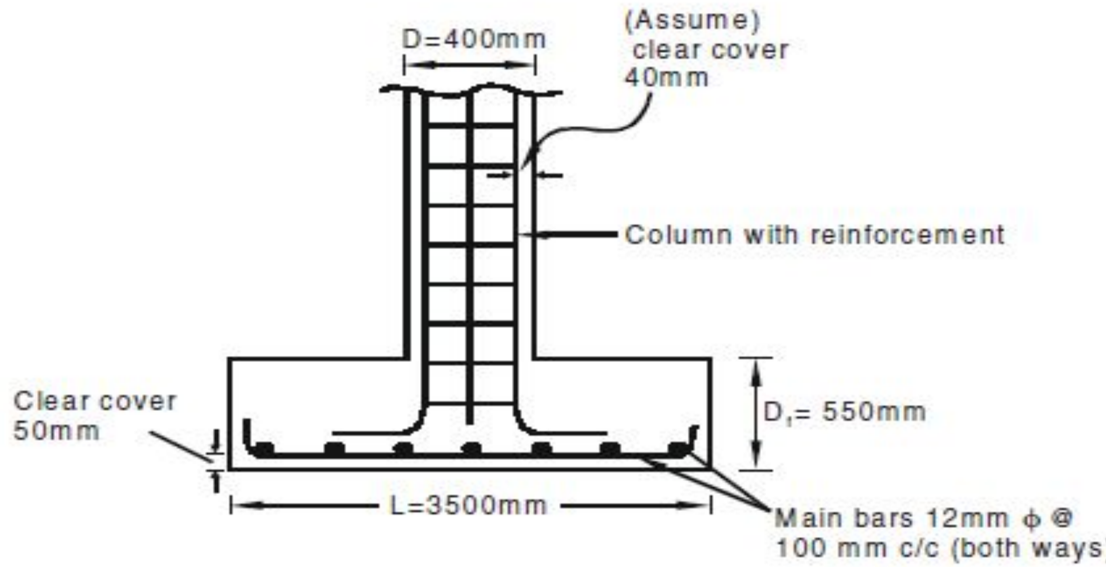
$$= 47 \times 12 = 564\text{mm}$$

Providing 50mm side cover

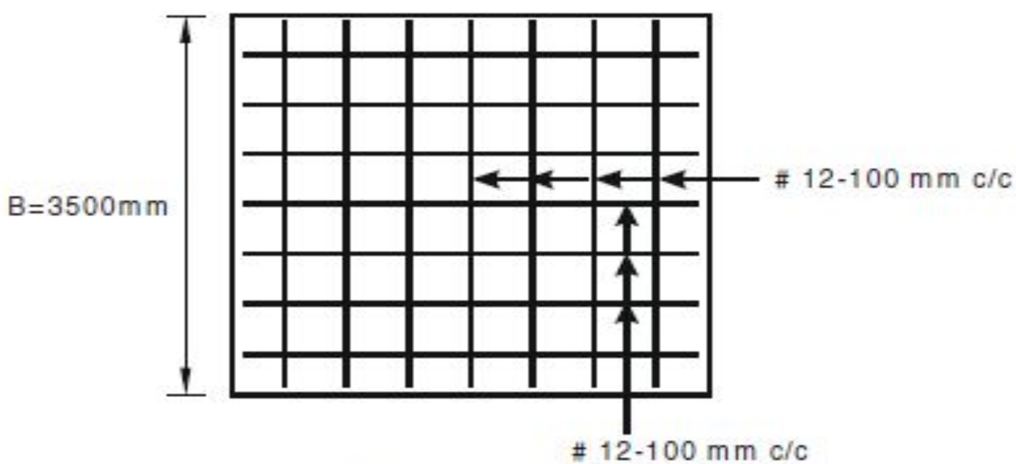
$$\therefore \text{Length available} = \frac{1}{2}[B - b] - 50 = \frac{1}{2}[3500 - 400] - 50 = 1500\text{mm} > L_d$$

Hence (ok)

Step 10 : Reinforcement details



L/S of footing



Plan of footing

2. **Design a Reinforced concrete footing for a rectangular column of section 300mm × 500mm supporting axial factored load of 1500 kN. The SBC of the soil at the site is 200 kN/m² (20 tons/m²). Use M20 concrete and Fe 415 steel.**

Solution :

Given $P_u = 1500\text{kN}$, $b = 300\text{mm} = 0.3\text{m}$, $D = 500\text{mm} = 0.5\text{m}$, $\text{SBC} = 200\text{kN/m}^2$, $f_{ck} = 20$ N/mm² and $f_y = 415$ N/mm²

Service load or working load, $P = \dfrac{P_u}{1.5} = \dfrac{1500}{1.5} = 1000\text{kN}$

Step 1 : Load calculations

Column axial load, $P = 1000\text{kN}$

Self weight of footing $= 0.1 \times 1000 = 100\text{kN}$

(Assume 10% of column load)

$\therefore$ Total load, $W = 1100\text{kN}$

Step 2 : Area of footing required

$$A_{\text{required}} = \frac{\text{Total load}}{\text{SBC of soil}} = \frac{W}{\text{SBC}} = \frac{1100}{200} = 5.5\text{m}^2$$

Step 3 : Size of footing and Area provided

For rectangular footing, size of footing $= L \times B$ $\quad | \, A = L \times B$

Assume one side, i.e., $B = 2\text{m}$

$$\therefore L = \frac{A}{B} = \frac{5.5}{2} = 2.75\text{m say } 3\text{m}$$

$\therefore$ provide 3m × 2m rectangular footing (i.e., $L = 3\text{m}$, $B = 2\text{m}$)

$\therefore$ Total area or Area provided, $A_{\text{provided}} = L \times B = 3 \times 2 = 6\text{m}^2$

Step 4 : Net upward pressure (q)

$$q = \frac{\text{Column axial load}}{\text{Area provided}} = \frac{1000}{6} = 166.66 \text{ kN/m}^2 < 200 \text{ kN/m}^2 \text{ (SBC)}$$

Hence (ok)

Step 5 : Bending moment calculation (M_u)

- **Along longer direction**

$$M_1 = q \times B \times \frac{1}{2}\left(\frac{L-D}{2}\right)^2$$

$$= 166.66 \times 2 \times \frac{1}{2}\left(\frac{3-0.5}{2}\right)^2$$

$$M_1 = 260.4 \text{kN.m}$$

Factored moment, $M_{u_1} = 1.5 \times M_1 = 1.5 \times 260.4 = 390.6$ kN.m

- **Along shorter direction**

$$M_2 = q \times L \times \frac{1}{2}\left(\frac{B-b}{2}\right)^2$$

$$= 166.66 \times 3 \times \frac{1}{2}\left(\frac{2-0.3}{2}\right)^2$$

$$M_2 = 180.61 \text{ kN.m}$$

Factored moment, $M_{u_2} = 1.5 \times M_2 = 1.5 \times 180.61 = 270.92$ kN.m

Step 6 : Effective depth required

Now, consider longer direction moment (Maximum moment)

Equating, $M_{u_1} = M_{u_{lim}}$ (Balanced section)

$$M_{u_{lim}} = 0.138 f_{ck} b d_f^2 \qquad | \text{ B = 2m =2000mm}$$

$$d_f = \sqrt{\frac{M_u}{0.138 f_{ck} B}} = \sqrt{\frac{390.6 \times 10^6}{0.138 \times 20 \times 2000}} = 266\text{mm}$$

Now increase footing depth (d_f) 1.75 to 2 times due to shear consideration

$$\therefore d_f = 2 \times 266 = 532\text{mm say } 550\text{mm}$$

Overall depth of footing, $D_f = d_f + d^1$ | Assume $d^1 = 50$mm

$$= 550 + 50$$

$$D_f = 600\text{mm}$$

Step 7 : Design of steel reinforcement or Area of steel reinforcement

By referring IS : 456 - 2000, G-1.1 (b)

- **Along longer direction**

$$M_{u_1} = 390.6 \times 10^6 \text{ N.mm}$$

$$\therefore M_{u_1} = 0.87\, f_y A_{st_1} d_f \left[1 - \left(\frac{f_y A_{st_1}}{f_{ck} B d_f}\right)\right] \qquad | \text{ Here, B = 2000mm}$$

$$390.6 \times 10^6 = 0.87 \times 415 \times 550\, A_{st_1} \left[1 - \left(\frac{415 A_{st_1}}{20 \times 2000 \times 550}\right)\right]$$

$$390.6 \times 10^6 = 198577.5\, A_{st_1} - 3.74\, A_{st_1}^2$$

$$\therefore 3.74\, A_{st_1}^2 - 198577.5\, A_{st_1} + 390.6 \times 10^6 = 0$$

By solving using quadratic equation, we get

$$A_{st_{1\text{required}}} = 2045.81\text{mm}^2$$

$$\text{Area of steel per meter} = \frac{2045.81}{B} = \frac{2045.81}{2} = 1022.90\text{mm}^2$$

Assume 12mm dia bars

$$\text{Area of one bar, } a_{st} = \frac{\pi \times 12^2}{4} = 113.09\text{mm}^2$$

$$\therefore \text{Spacing of reinforcement, } S = \frac{1000 a_{st}}{A_{st_1}} = \frac{1000 \times 113.09}{1022.90} = 110.55\text{mm}$$

∴ Provide 12mm dia bars at 100mm c/c along longer direction

$\left(A_{st_{1\text{provided}}} = 1130.9\text{mm}^2 \text{ per meter}\right)$

∴ Total $A_{st_{1\text{provided}}} = 2 \times 1130.9 = 2261.8\text{mm}^2 > A_{st_{1\text{required}}}$

Hence (ok)

- **Along shorter direction**

$$M_{u_2} = 270.92 \times 10^6 \text{N.mm}$$

$$M_{u_2} = 0.87\, f_y A_{st_2} d_f \left[1 - \left(\frac{f_y A_{st_2}}{f_{ck} L d_f}\right)\right] \quad | \text{ Here L} = 3000\text{m}$$

$$270.92 \times 10^6 = 0.87 \times 415 \times 550\; A_{st_2} \left[1 - \left(\frac{415 A_{st_2}}{20 \times 3000 \times 550}\right)\right]$$

$$270.92 \times 10^6 = 198577.5\; A_{st_2} - 2.49\; A_{st_2}^2$$

$$\therefore 2.49\; A_{st_2}^2 - 198577.5\; A_{st_2} + 270.92 \times 10^6 = 0$$

By solving quadratic equation, we get

$$A_{st_{2_{required}}} = 1388.47\text{mm}^2$$

$$\text{Area of steel per meter} = \frac{1388.47}{L} = \frac{1388.47}{3} = 462.82\text{mm}^2$$

Assume 10mm dia bars

$$\text{Area of one bar, } a_{st} = \frac{\pi \times 10^2}{4} = 78.53\text{mm}^2$$

$$\therefore \text{Spacing of reinforcement, } S = \frac{1000a_{st}}{A_{st_2}} = \frac{1000 \times 78.53}{462.82} = 169.69\text{mm}$$

$\therefore$ Provide 10mm dia bars at 150mm c/c along shorter direction

$$\left(A_{st_{2_{provided}}} = 523.53\text{mm}^2 \text{ per meter} \right)$$

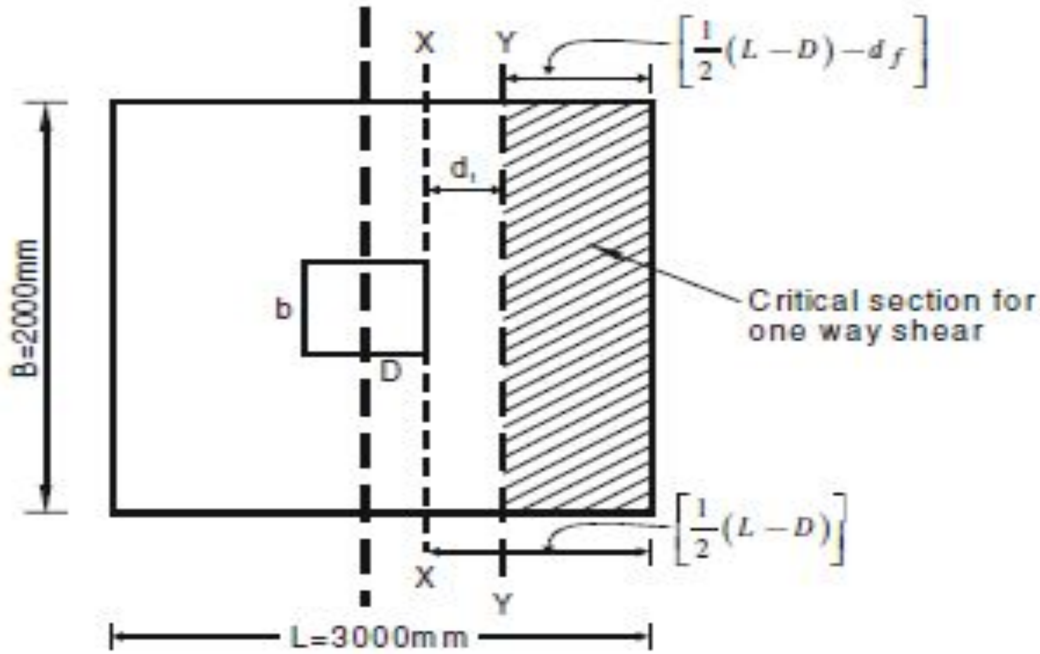
$$\therefore \text{Total } A_{st_{2_{provided}}} = 3 \times 523.53 = 1570.6\text{mm}^2 > A_{st_{2_{required}}}$$

Hence (ok)

Step 8 : Design for shear

1. Check for one way shear

The critical section is taken at a distance of 'd_f' away from the face of column.



b=300mm = 0.3m
D=500mm = 0.5m

∴ Shear force at the critical section is given by

➢ **Along longer direction,**

$$V = q \times B \times \text{critical section for one way shear}$$

$$= q \times B \times \left[\frac{1}{2}(L-D)-d_f\right]$$

$$= 166.66 \times 2 \times \left[\frac{1}{2}(3-0.5)-0.55\right]$$

$$V = 233.32\text{kN}$$

- **Along shorter direction,**

$$= q \times L \times \left[\frac{1}{2}(B-b)-d_f\right]$$

$$= 166.66 \times 3 \times \left[\frac{1}{2}(2-0.3)-0.55\right]$$

$$V = 150\text{kN}$$

$\therefore$ Shear force in longer direction is more

$$\text{i.e., } V = 233.32\text{kN}$$

- Factored shear force $V_u = 1.5 \times V = 1.5 \times 233.32 = 350\text{kN}$
- Nominal shear stress, $\tau_v = \dfrac{V_u}{Bd_f} = \dfrac{350 \times 10^3}{2000 \times 550} = 0.31\text{N/mm}^2$
- Percentage of steel, $p_t = \dfrac{100A_{st_1}}{Bd_f} = \dfrac{100 \times 2261.8}{2000 \times 550} = 0.2\%$
- Design of shear strength of concrete, τ_c

  By referring IS : 456 - 2000 Table No. 19. Using $f_{ck} = 20\text{N/mm}^2$, $p_t = 0.2\%$

$$\tau_c = 0.32\ \text{N/mm}^2 \text{ (By using interpolation)}$$

- Permissible shear stress $= k\tau_c$

$$k = 1$$

$$\Rightarrow k\tau_c = 1 \times 0.32 = 0.32\ \text{N/mm}^2$$

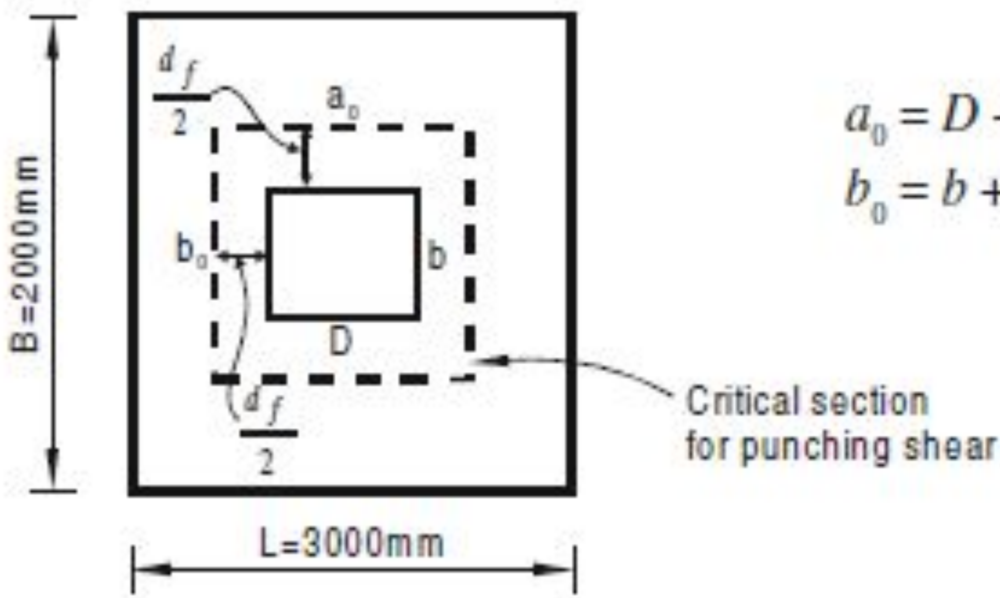
➢ Comparisons

$k\tau_c > \tau_v$ — Design is safe against one way shear

Hence stresses are within permissible limits

2. Check for two way shear or punching shear

The critical section is taken at a distance $\frac{d_f}{2}$ or 0.5 d_f away from the face of column.



$a_0 = D + d_f = 0.5 + 0.55 = 1.05$m or 1050 mm

$b_0 = b + d_f = 0.3 + 0.55 = 0.85$m or 850 mm

∴ Shear force at the critical section is given by

$$V = q \times [A_{prov} - a_o \times b_o] \qquad |A_{provided}$$

$$= 166.66 \times [6 - 1.05 \times 0.85]$$

$$\therefore V = 851.21 \text{kN}$$

- Factored shear force $V_u = 1.5 \times V = 1.5 \times 851.21 = 1276.82$kN

- Nominal shear stress, $\tau_v = \dfrac{V_u}{2(a_o + b_o) \times d_f} = \dfrac{1276.82 \times 10^3}{2(1050 + 850) \times 550} = 0.61 \text{N/mm}^2$

- Design of shear strength of concrete, τ_c

$$\tau_c = 0.25\sqrt{f_{ck}} \text{ — from IS : 456 - 2000, clause 31.6.3.1}$$

$$\tau_c = 0.25\sqrt{20} = 1.11 \text{ N/mm}^2$$

- Permissible shear stress = $k_s \tau_c$

$$k_s = (0.5 + \beta_c) \not> 1$$

$$\text{where, } \beta_c = \frac{\text{Short side of column}}{\text{Long side of column}} = \frac{b}{D} = \frac{0.3}{0.5} = 0.6$$

$$\Rightarrow k_s = (0.5 + 0.6) = 1.1 \not> 1$$

$$k_s = 1$$

- Comparisons

  $k_s \tau_c = 1 \times 1.11 = 1.11 \text{ N/mm}^2 > \tau_v$

  Hence Design is safe against two way shear

Step 9 : Check for development length of bar

Refer IS : 456 - 2000, clause 26.2.1

$$L_d = \frac{\phi \sigma_s}{4\tau_{bd}} \qquad \mid \phi = 12\text{mm (main bar dia)}$$

$$L_d = 47\phi \text{ — for Fe 415}$$

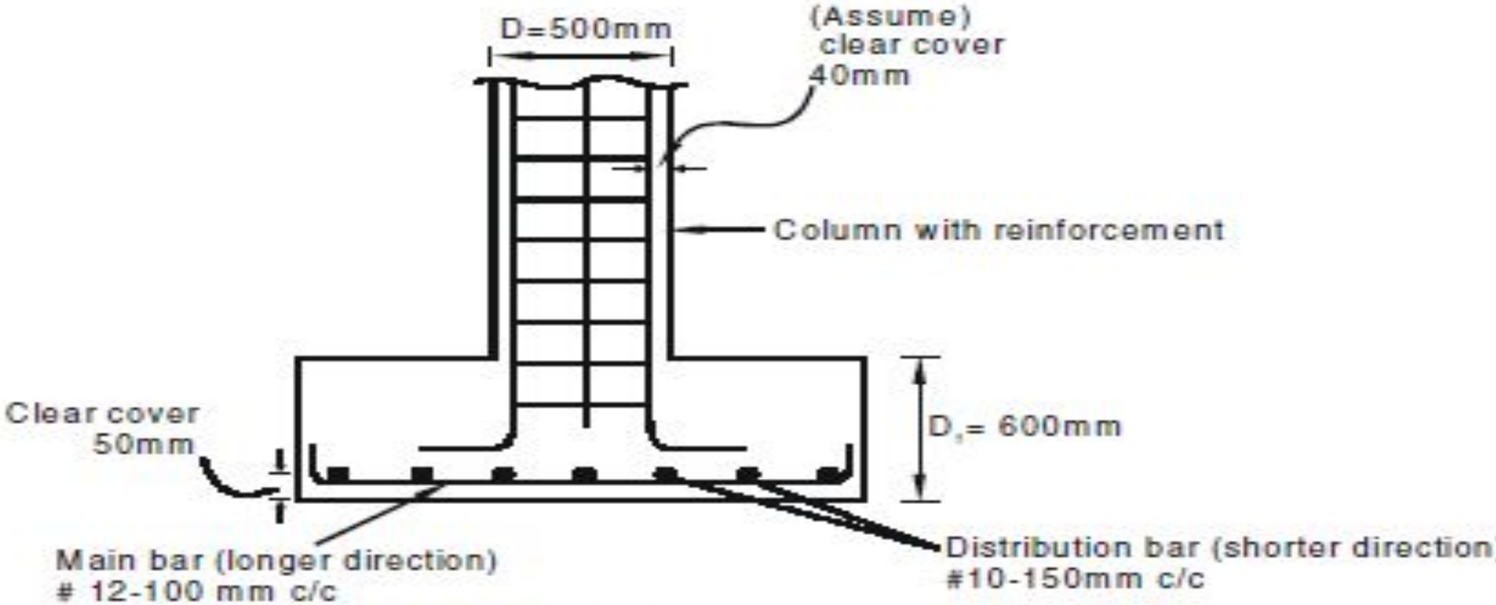
$$= 47 \times 12 = 564\text{mm}$$

Providing 50mm side cover

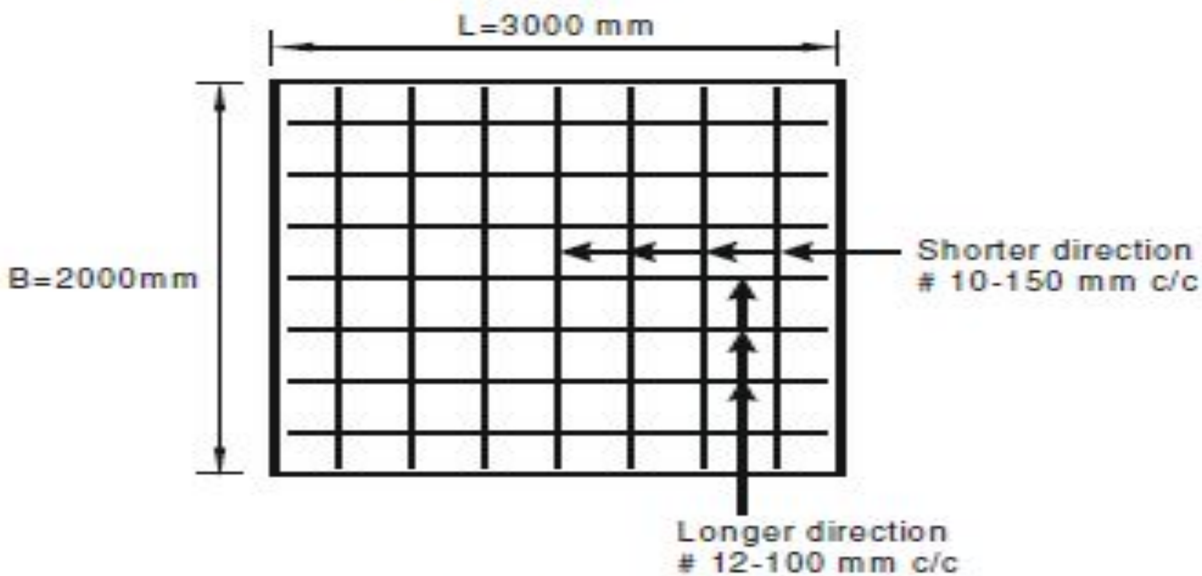
$$\therefore \text{Length available} = \frac{1}{2}[B - b] - 50 = \frac{1}{2}[2000 - 300] - 50 = 800\text{mm} > L_d$$

Hence (ok)

Step 10 : Reinforcement details



L/S of footing



Plan of footing

DESIGN OF ISOLATED RECTANGULAR AND SQUARE FOOTINGS FOR AXIAL LOAD AND MOMENT

Walls or columns often transfer moments as well as vertical loads to their footings. These moments may be the result of gravity loads. Footings are often subjected to lateral loads or overturning moments, in addition to vertical loads. These types of loads are typically seismic or wind loads. Such a situation is represented by the vertical load '*P*' and the bending moment *M*.

The load '*P*' acting on a footing may act eccentrically w.r.t the centroid of the footing base. The eccentricity '*e*' may result from column transmitting a moment 'M' in addition to the vertical load as shown in Fig 5.17.

Combined axial and bending moment increases the pressure one edge or corner of a footing.

$$e = \frac{M}{P}$$

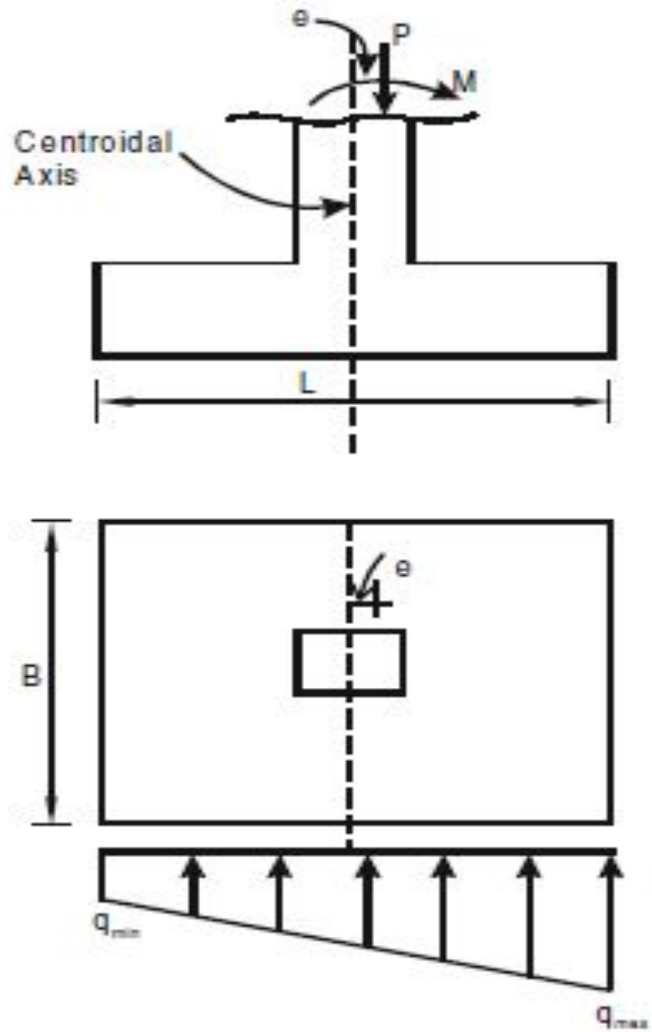


Fig. 5.17 : Column subjected to axial load and bending

The soil pressure due to axial force is uniform and is given by, $q = \frac{P}{A}$.

Due to moment, one side tension is created and the other side compression is created. The maximum and minimum soil pressure are computed with the help of compression and tension.

$$\therefore \text{Maximum soil pressure, } q_{max} = \frac{P}{A} + \frac{M}{Z}$$

$$\text{Minimum soil pressure, } q_{min} = \frac{P}{A} - \frac{M}{Z}$$

The total pressure diagram is shown is Fig. 5.17, which varies linearly from q_{min} to q_{max}.

The average soil pressure is calculated using q_{max} and q_{min}

$$\text{i.e., } q_{average} = \frac{q_{max} + q_{min}}{2}$$

Design bending moment and Design of one way shear are calculated using q_{min} and q_{max}. Design of two way shear is calculated with the help of $q_{average}$

WORKED EXAMPLES ON DESIGN OF ISOLATED FOOTINGS WITH AXIAL LOAD AND MOMENT

1. Design an isolated footing for an RC column of size 300 × 300mm which carries a vertical load of 800kN together with an uniaxial moment of 40 kN.m. The SBC of soil is 250 kN/m². Use M25 grade concrete and Fe 415 grade steel.

Solution :

Given : $b = 300\text{mm}$, $D = 300\text{mm}$, $P = 800\text{kN}$, $M = 40\text{kN.m}$

SBC = 250kN/m^2, $f_{ck} = 25\text{N/mm}^2$ and $f_y = 415\text{N/mm}^2$

Step : 1 Load calculation

Column axial load, $P = 800$ kN

Self weight of footing $= 0.1 \times 800 = 80$ kN

(Assume 10% of column load)

$\therefore$ Total load, $W = 880$ kN

Step : 2 Area of footing required

$$A_{\text{required}} = \frac{\text{Total load}}{\text{SBC of soil}} = \frac{W}{SBC} = \frac{880}{250} = 3.52\text{m}^2$$

Step : 3 Size of footing and Area provided

Let us provide square isolated footing

$$\text{Size of footing, } L = B = \sqrt{A_{\text{required}}} = \sqrt{3.52} = 1.87\text{m}$$

$\therefore$ Provide 2m × 2m square footing

$\therefore L = B = 2\text{m}$

$\therefore$ Total area provided, $A_{\text{provided}} = L \times B = 2 \times 2 = 4\text{m}^2$

Step : 4 Net upward pressure (q)

For Axial load and Bending moment

$$q = \frac{P}{A_{prov}} \pm \frac{M}{Z}$$

L = Length of footing

B = Breadth of footing

$$\text{Here, } Z = \frac{I}{L/2} = \frac{\frac{LB^3}{12}}{L/2} = \frac{LB^3}{12} \times \frac{2}{L}$$

$$\therefore \; Z = \frac{B^3}{6}$$

$$\Rightarrow \; q_{max} = \frac{P}{A_{prov}} + \frac{M}{\frac{B^3}{6}}$$

$$= \frac{P}{A_{prov}} + \frac{6M}{B^3}$$

$$q_{max} = \frac{800}{4} + \frac{6 \times 40}{(2)^3} = 200 + 30 = 230 \text{ kN/m}^2 < 250 \text{ kN/m}^2 \text{ (SBC)}$$

$$\text{Now } q_{min} = \frac{800}{4} - \frac{6 \times 40}{(2)^3} = 200 - 30 = 170 \text{ kN/m}^2 > 0$$

Hence (ok)

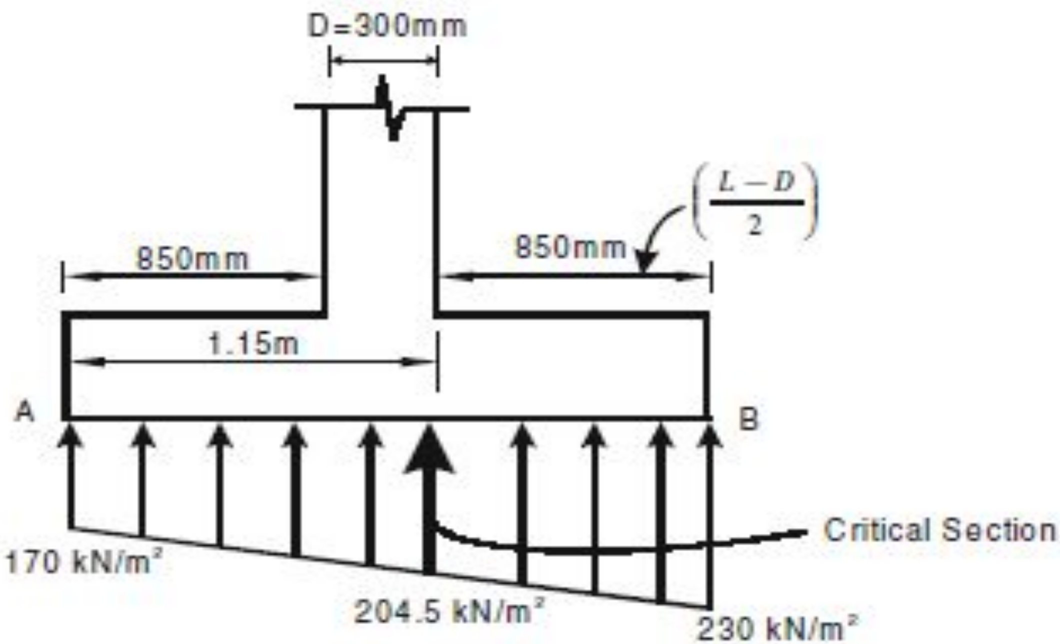
Hence upward pressure of soil are

$$q_{max} = 230 \text{ kN/m}^2$$
$$q_{min} = 170 \text{ kN/m}^2$$

$\therefore$ Average pressure at the centre of the footing, $q_{average} = \dfrac{q_{max} + q_{min}}{2}$

$$q_{average} = \frac{230 + 170}{2} = 200 \text{ kN/m}^2$$

Step : 5 Bending moment calculation (M_u)



$$\frac{L-D}{2} = \frac{2000-300}{2}$$
$$= 850\text{mm}$$

Now using interpolation method

For, 0m length, $q = 170$ kN/m²

1.15m length, $q = ?$

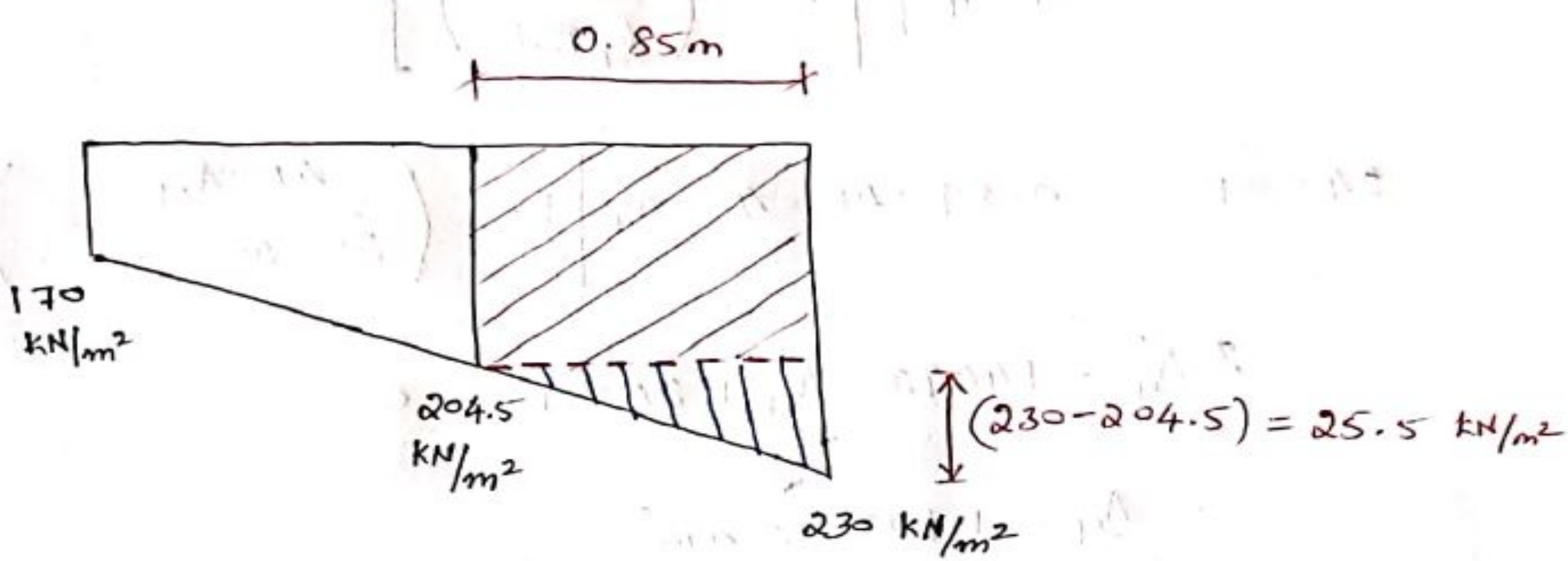
2m length, $q = 230$ kN/m²

$\therefore q = 204.5$ kN/m² @ 1.15m length form 'A'

Upward pressure of soil at critical section, $q = 204.5$ kN/m²

$\therefore$ Bending moment at the critical section in the footing is

M = (Total force under cantilever) * (Distance of its centroid from column face)



[(Rectangular portion) + (Triangular portion)] × Width of footing

$$\therefore M = \left[\left(204.5 \times 0.85 \times \frac{0.85}{2}\right) + \left(\frac{1}{2} \times 0.85 \times 25.5 \times \frac{2}{3} \times 0.85\right)\right] \times B$$

$$M = [(73.87 + 6.141)] \times 2 = 160.02 \text{ KN.m}$$

$\therefore$ Factored moment, $M_u = 1.5 \times M = 1.5 \times 160.02 = 240.03$ KN.m

Step 6: Effective depth required

$$d_f = \sqrt{\frac{M_u}{0.138 f_{ck} B}} = \sqrt{\frac{240.03 \times 10^6}{0.138 \times 25 \times 2000}} = 186.51 \text{ mm}$$

Now increase footing depth (d_f) due to shear consideration

$\therefore d_f = 2 \times 186.51 = 373.02$ mm , Say $d_f = 400$ mm

Assume $d' = 50$ mm

$\therefore$ Overall depth of footing, $D_f = d_f + d' = 400 + 50$

$\therefore D_f = 450$ mm

Step: 7 **Design of Steel reinforcement or Area of Steel reinforcement**

$$M_u = 0.87 f_y A_{st} d_f \left[1 - \left(\frac{f_y A_{st}}{f_{ck} B d_f}\right)\right]$$

$$240 \times 10^6 = 0.87 \times 415 \times 400 A_{st} \left[1 - \left(\frac{415 A_{st}}{25 \times 2000 \times 400}\right)\right]$$

$$3 A_{st}^2 - 144420 A_{st} + 240 \times 10^6 = 0$$

$$\therefore A_{st} = 1723.52 \text{ mm}^2$$

Area of steel per meter $= \frac{1723.52}{B} = \frac{1723.52}{2} = 861.76 \text{ mm}^2$

Assume 12 mm ϕ bars

Area of one bar, $a_{st} = \frac{\pi}{4} \times 12^2 = 113.09 \ mm^2$

$\therefore$ Spacing of reinforcement, $S = \frac{1000\, a_{st}}{A_{st}} = \frac{1000 \times 113.09}{861.76}$

$= 131.23 \ mm$

$\therefore$ Provide 12mm dia bars at 125 mm c/c along shorter and longer direction (both ways)

$A_{st_{provided}} = \frac{1000 \times 113.09}{125} = 904.72 > 861.76 \ mm^2$

Hence (OK)

Total $A_{st_{provided}} = B \times 904.72 = 2 \times 904.72 = 1809.44 \ mm^2$

* Factored Shear Force, $V_u = 1.5 \times V = 1.5 \times 200.92 = 301.38$ kN

* Nominal Shear Stress, $\tau_v = \dfrac{V_u}{B d_f} = \dfrac{301.38 \times 10^3}{2000 \times 400} = 0.37 \ N/mm^2$

* Percentage of steel, $p_t = \dfrac{100 A_{st}}{B d_f} = \dfrac{100 \times 1809.44}{2000 \times 400} = 0.22 \ \%$

* Design of shear strength of concrete, τ_c

  Now, $p_t = 0.22\%$ and $f_{ck} = 25 \ N/mm^2$ using IS:456-2000

  Table-19

  $\tau_c = 0.339 \ N/mm^2$ (Using interpolation method)

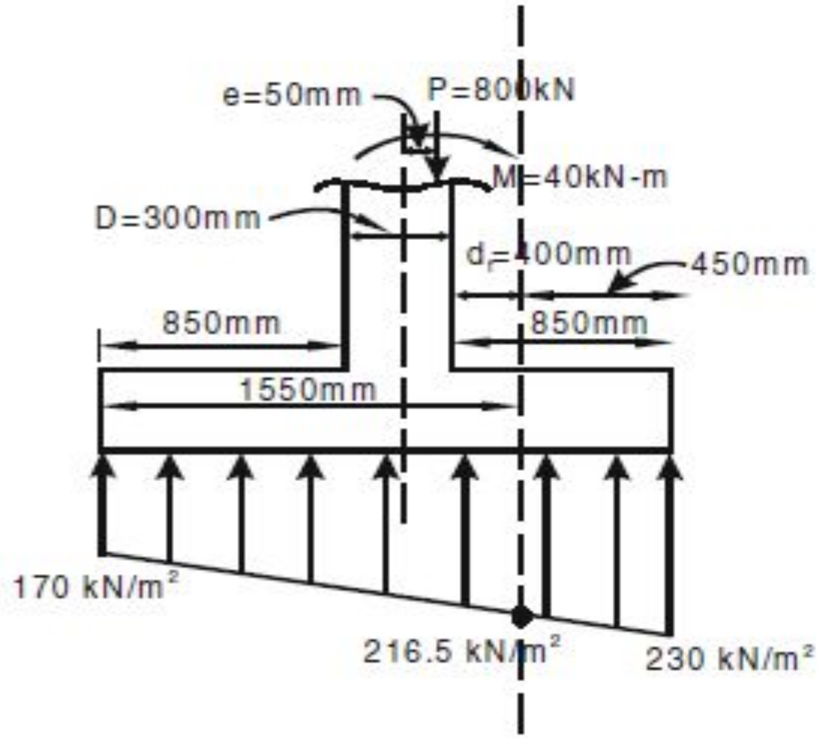
* Permissible shear stress $= k \tau_c$

  Refer IS: 456-2000, clause 40.2.1.1

Step : 8 Design for shear

1. Check for one way shear

The critical section is taken at a distance of 'd_f' away from the face of column.



$$M = 40\text{kN.m}$$

$$P = 800\text{kN}$$

$$e = \frac{M}{P} = \frac{40}{800} = 0.5\text{m}$$

$$\therefore e = 50\text{mm}$$

$$\frac{L-D}{2} = \frac{2000-300}{2} = 850\text{mm}$$

$\therefore$ Shear force at the critical section is given by

$$V = [\text{Total force}] \times B \times \left[\frac{1}{2}(L-D) - d_f\right]$$

$$V = \left[\frac{230+216.5}{2}\right] \times 2 \times [0.85 - 0.4] = 200.92\text{kN}$$

* Factored Shear Force, $V_u = 1.5 \times V = 1.5 \times 200.92 = 301.38$ kN

* Nominal Shear Stress, $\tau_v = \frac{V_u}{B d_f} = \frac{301.38 \times 10^3}{2000 \times 400} = 0.37\ N/mm^2$

* Percentage of steel, $p_t = \frac{100 A_{st}}{B d_f} = \frac{100 \times 1809.44}{2000 \times 400} = 0.22\ \%$

* Design of shear strength of concrete, τ_c

Now, $p_t = 0.22\%$ and $f_{ck} = 25\ N/mm^2$ using IS: 456-2000

Table-19

$\tau_c = 0.339\ N/mm^2$ (Using interpolation method)

* Permissible shear stress $= k\tau_c$

Activa
Go to S

Refer IS : 456-2000, clause 40.2.1.1

$k = 1$ (∵ $k = 1$ for overall depth of footing more than 300 mm)

$$\Rightarrow k\tau_c = 1 \times 0.339 = 0.339 \ N/mm^2$$

* Comparisons

$k\tau_c < \tau_v$ — Design is not safe against one way shear

Now Assume $p_t = 0.3\% > p_t = 0.22\%$

$\therefore \tau_c = 0.38 \ N/mm^2$ (Using interpolation method)

$$\Rightarrow k\tau_c = 0.38 > \tau_v \ (0.37 \ N/mm^2)$$

– Hence Design is Safe against One way shear

Now Recalculate the $A_{st_{provided}}$

$p_t = 0.3\ \%$

$$A_{st} = \frac{p_t B d_f}{100}.$$ consider $B = 1$ m width

$$= \frac{0.3 \times 1000 \times 400}{100} = 1200\ mm^2$$

Assume 16 mm ϕ bars

$\therefore$ Spacing of reinforcement, $S = \frac{1000\ a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 16^2}{1200}$

$= 167.5$ mm

∴ Provide 16 mm ϕ bars @ 150 mm c/c along shorter and longer direction (both ways)

$$\therefore \; A_{st\,provided} = \frac{1000 \times \frac{\pi}{4} \times 16^2}{150} = 1340.41 \text{ mm}^2 > 1200 \text{ mm}^2$$

$$\text{Total } A_{st\,provided} = B \times 1340.41$$

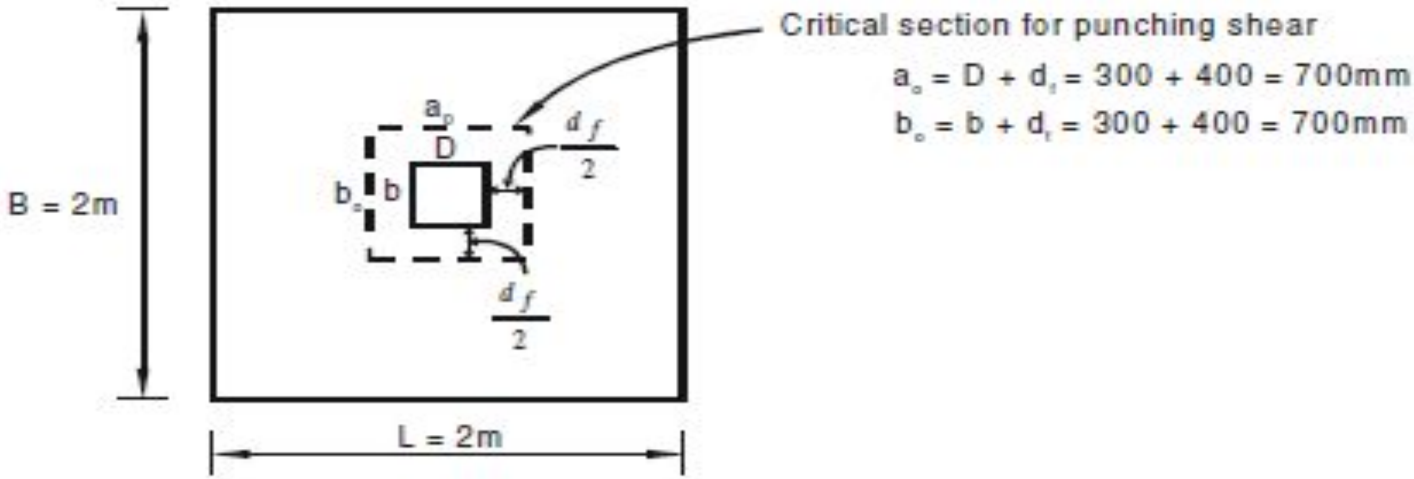
$$= 2 \times 1340.41$$

$$A_{st\,provided} = 2680.82 \text{ mm}^2$$

Hence (OK)

2. Check for two way shear

The critical section is taken at a distance $\frac{d_f}{2}$ away from the face of column



$\therefore$ Shear force at the critical section is given by

$$V = \text{average pressure} \times [A_{\text{provided}} - a_o \times b_o]$$
$$= q_{\text{average}} \times [A_{\text{provided}} - a_o \times b_o]$$
$$= 200 \times [4 - 0.7 \times 0.7]$$
$$V = 702\text{kN}$$

- Factored shear force, $V_u = 1.5 \times V = 1.5 \times 702 = 1053$ kN

- Nominal shear stress, $\tau_v = \dfrac{V_u}{2(a_o + b_o) \times d_f} = \dfrac{1053 \times 10^3}{2(700 + 700) \times 400} = 0.94$ N/mm²

- Design shear strength of concrete, τ_c

  $\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25 \text{N/mm}^2$

- Permissible shear stress = $k_s \tau_c$

$$k_s = (0.5 + \beta_c) \ngtr 1$$

$$\beta_c = \frac{b}{D} = \frac{0.3}{0.3} = 1$$

$$\therefore k_s = (0.5 + 1) = 1.5 \ngtr 1$$

$$\Rightarrow k_s = 1$$

- Comparisons

  $k_s \tau_c = 1 \times 1.25 = 1.25 \text{ N/mm}^2 > \tau_v$

  Hence Design is safe against two way shear

Step : 9 Check for development length of bars

Refer IS : 456-2000, Clause 26.2.1

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

For Fe 415 steel, $L_d = 47\ \phi$ $\qquad$ $\phi = 16\text{mm}$

$$\therefore L_d = 47 \times 16 = 752 \text{ mm}$$

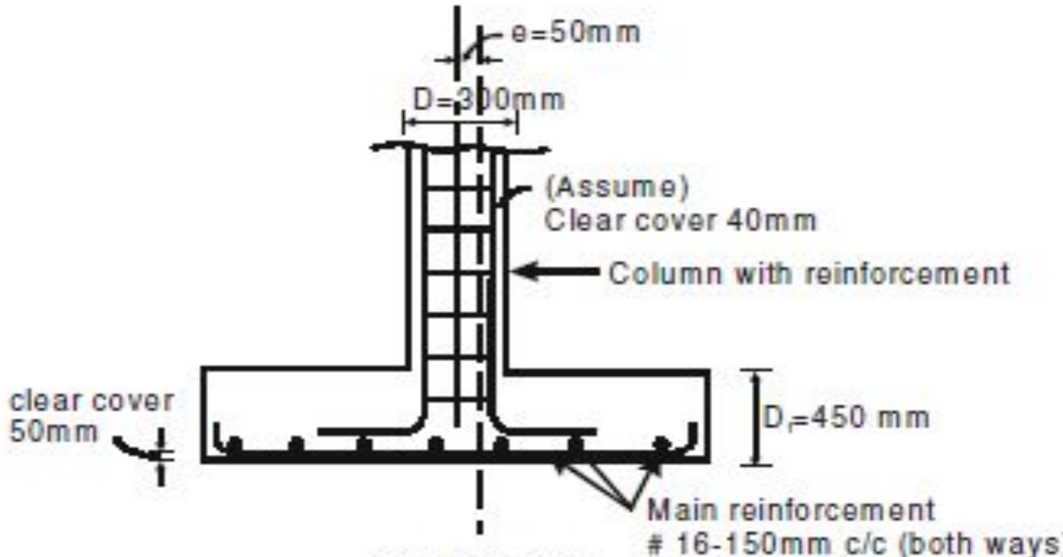
Providing 50mm side cover

$$\therefore \text{Length available } = \frac{1}{2}[B - b] - 50 = \frac{1}{2}[2000 - 300] - 50$$

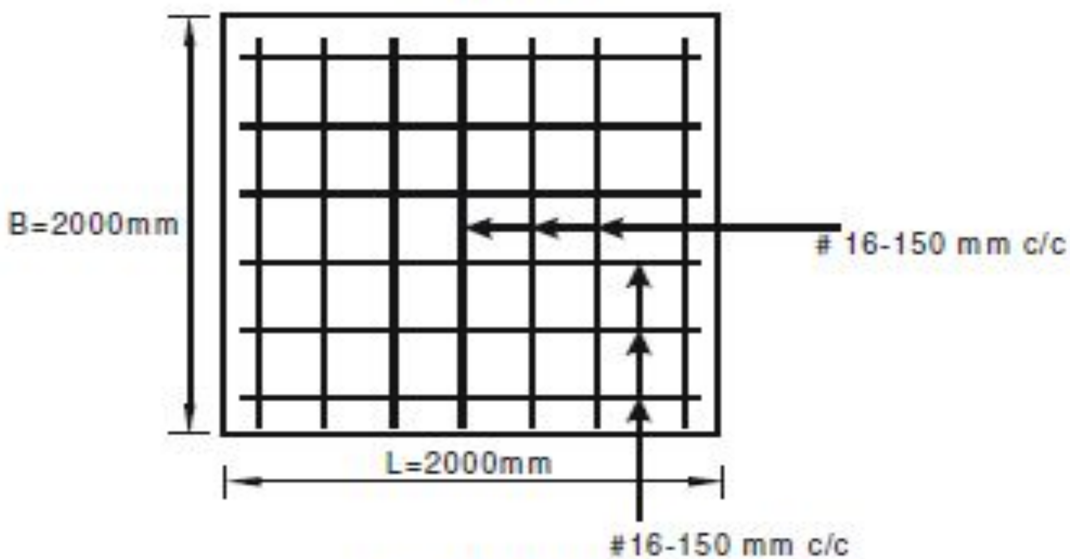
$$= 800\text{mm} > L_d$$

Hence (ok)

Step : 10 Reinforcement details



L/S of footing



Plan of footing

Textbooks:

1. Unnikrishnan Pillai and Devdas Menon, " Reinforced Concrete Design" , McGraw Hill, New Delhi
2. Subramanian, " Design of Concrete Structures" , Oxford university Press
3. H J Shah, "Reinforced Concrete Vol. 1 (Elementary Reinforced Concrete)" , Charotar Publishing House Pvt. Ltd.

Reference Books:

1. P C Varghese, "Limit State design of reinforced concrete" , PHI, New Delhi.
2. W H Mosley, R Husle, J H Bungey, "Reinforced Concrete Design", MacMillan Education, Palgrave publishers.
3. Kong and Evans, "Reinforced and Pre-Stressed Concrete", Springer Publications.
4. A W Beeby and Narayan R S, "Introduction to Design for Civil Engineers", CRC Press.
5. Robert Park and Thomas Paulay, "Reinforced Concrete Structures", John Wiley & Sons, Inc.

THANK YOU